Deconstruction in Mathematics Education

The Eight Missing Links of Mandarin Mathematics

Allan Tarp, the MATH**e**CADEMY.net, June 2012

This YouTube video on deconstruction in mathematics education describes how natural mathematics is made difficult by removing eight links to its roots, the natural fact Many. The missing links make mathematics a privilege to a mandarin class wanting to monopolize public offices. Reopening the eight missing links will make mathematics easy and accessible to all.

Session I: Primary school, http://youtu.be/sTJiQEOTpAM

Screen 2 & 3

Bo: Welcome to the MATHeCADEMY.net Channel. And welcome to our series on deconstruction in mathematics education. My name is Bo. Today we address the question "Mathematics IS hard - or is it?" We begin with primary school. And welcome to our guest, Allan, who uses deconstruction in his work.

Allan: Thank you, Bo.

Bo: Allan, what does deconstruction mean to you?

Allan: Well Bo. To me, deconstruction means what it says, destruction and reconstruction. It is the method that is used by postmodern skeptical thinking that dates back to the ancient Greek sophists. The sophists warned against patronization that is hidden in choices that are presented as nature. So to avoid hidden patronization, false nature must be unmasked as choice. And to deconstruct then means to discover or to invent alternatives to choices that are presented as nature.

Bo: Thank you, Allan. Do you have a short answer to today's question?

Allan: To me, the short answer to the question "Mathematics IS hard - or is it" is that mathematics is not hard by nature, but by choice. Mathematics is made difficult by its missing links.

Bo: And what do you mean by a missing link?

Screen 4

Allan: I use the term missing link to indicate that mathematics has lost its links to its root, the natural fact Many. However, if these links are established again, then mathematics will once more become a science about Many. And accepting it as a natural science will make mathematics easy and accessible to all, instead of difficult and accessible only to few.

Bo: So Allan, what is the first missing link?

Screen 5

Allan: Well Bo, the first missing link is the total. To deal with many, we total, i.e. we ask the question "how many?" And the answer is given by the total. That is why we use the word algebra that means to unite in Arabic. Thus all mathematics statements should begin with its subject, the total, and specify what the total is. The first step is to represent the total in three ways, by physical sticks, by graphical strokes, and by spoken words. Thus four things can be represented by four sticks on a table; and by four strokes on a paper; and by the word four. Now we can write that T equals stroke stroke stroke stroke.

Bo: That is what the Romans did, isn't it?

Allan: Indeed it is. But the Arabs went one step further by using icons. They united the four strokes into one single symbol consisting of four strokes. Thus they transformed four ones into one fours.

Bo: But isn't four ones and one fours the same?

Screen 6

Allan: Well Bo. Four ones means that you count in ones, so that one is the unit. Whereas one fours means that you count in fours, so that four is the unit. And that you count by bundling and stacking the total in fours. Thus rearranging sticks and strokes into icons is the second missing link. If we write the digits in a less sloppy way, we can see that there are five sticks and strokes in the five-icon, and six strokes in the six-icon, etc.

Screen 7

Bo: But why do the icons stop at ten?

Screen 8

Allan: We count by bundling, but we never use the bundle-icon. If we count or bundle in fives, we count one, two, three, four, bundle, one bundle one, one bundle two etc. Or in a shorter way: 1, 2, 3, 4, 10, 11, 12 etc. Thus we do not use the five-icon. Likewise, ten does not need an icon when counting in tens. In this way ten is the only number with its own name but without its own icon. This makes ten a cognitive bomb since ten is the follower of nine while one zero is the follower of four when counting in fives. So ten is not one zero by nature, but by choice of the bundle-number.

Bo: So what is the third missing link?

Screen 9

Allan: The third missing link is cup-writing. First we count the sticks or strokes by bundling them in e.g. fours. Then we can place the bundles in a left bundle-cup, and the unbundled in a right single-cup. We don't have to place the physical bundles. For each bundle we just place a stick in the bundle-cup. Now we can use the icons to write the total using a decimal point to separate the bundles from the unbundled. In this way a total of 6 1s can be counted in 4s as 1 4s and 2 1s, and written as 1.2 4s. So a natural number includes a decimal point to separate the bundles from the unbundled, and a unit.

Bo: And what is the fourth missing link?

Screen 10

Allan: The fourth missing link is recounting, or changing the unit. Thus a total of 7 1s can be recounted in 5s as 1.2 5s, or as 1.3 4s, or as 2.1 3s, etc. If counted in tens, 7 1s become 0.7 tens. However, we only write 7 leaving out the unit and misplacing the decimal point one place to the right. This may be OK in business, but it creates learning problems in school.

Bo: I guess that recounting is done by de-bundling and re-bundling?

Screen 11

Allan: Yes. Re-counting can be done manually by de-bundling and re-bundling. However, counting in fours means to repeat taking away four, i.e. dividing the total by four. So the counting result can be predicted by a recount-formula saying that the total T can be bundled in bs T/b times. So T is (T/b) bs, which is the same as (T/b) times b. Thus the recount-formula predicts that recounting a total of 8 1s in 4s gives 8/4 of the 4s, which is 2 4s.

Bo: So recounting is another word for shifting units?

Screen 12

Allan: Yes. To shift the unit means to recount. The tradition calls it proportionality or linearity. Shifting unit is the core of mathematics and easy to learn as recounting. However, the tradition avoids recounting. Instead it insists that counting only takes place in tens, that the decimal point is misplaced and that the unit is left out and. Thus what is called natural numbers are in reality highly unnatural. Many learning problems may disappear by respecting that, of course, any natural number has a unit and a decimal point. And by postponing counting in tens as long as possible in grade one.

Bo: And then what is the fifth missing link?

Screen13 & 14

Allan: The fifth missing link is predicting numbers by calculations. To recount 4 5s in 6s manually, first we must count up the 4 5s. Then we must debundle them in 1s. And finally we must rebundle them in 6s. This is a long and tiresome job. However, if we use the recount-formula and a calculator, we can predict the result to be 3 6s and a rest. And the rest is found by removing the 3 6s from the 4 5s. So the result of recounting can be predicted by a calculator using division and subtraction. Pre-dicting recount results shows the very essence of mathematics. Mathematics is our language for num-ber-prediction. Experiencing its predicting ability yourself may remove many motivation problems.

Bo: Allan, you say that mathematics is a language?

Screen 15

Allan: Yes Bo. We describe the world by words and numbers, so we have two languages. Our word-language combines letters to words and words to statements that describe the world. Our number-language combines digits to numbers and numbers and operations to formulas that predict the world. However, a language also has a meta-language, a grammar. A language is like a two level house, a language-house: At the lower level the language describes the outside world. And at the upper level the grammar describes the language. Meeting a language before its grammar makes it easy to learn, as in the case of the word-language. Meeting a language after its grammar makes it difficult to learn, as in the case of the number-language, which claims that its grammar, mathematics, must be learned before it can be applied.

Bo: But clearly, to apply mathematics, you must first learn mathematics?

Allan: Here we are seduced by our words. If we talk about rooting instead of applying, the logic is the opposite: Of course the root comes before what it roots. And the roots of mathematics can be seen in the names algebra and geometry meaning earth-measuring in Greek and uniting in Arabic. So presenting a grammar before its language is not nature, but choice.

Bo: Allan, can you give more examples on how mathematics predicts?

Screen 16

Allan: Certainly, Bo. As a matter of fact, operations are invented to predict numbers. Thus 3+5 predicts the result of counting-on 5 times from 3. And 3 times 5 predicts the result of adding with 3 5 times. And 3 to the power of 5 predicts the result of multiplying with 3 5 times. Furthermore, we saw that digits are icons that contain the number of strokes they represent. Likewise, operations are also icons that show the counting processes they represent.

Bo: Can you please specify that?

Screen 17

Allan: I would like to. Taking away 4 is iconized as a horizontal stroke showing the trace left when dragging away the 4. Taking away 4 many times, i.e., taking away 4s, is iconized as an up-hill stroke showing the broom sweeping away the 4s. Placing a stack of 4 singles next-to another stack is iconized as a cross showing the juxtaposition of the two stacks. And building up a stack of 3 4s is iconized as an up-hill cross showing a 3 times lifting of the 4s.

Bo: From the recount-formula it seems as if division and multiplication come before addition and subtraction?

Screen 18

Allan: Indeed they do. After recounting is predicted by a rebundle-formula using division and multiplication, we can predict the unbundled by a restack-formula using subtraction and addition. So the natural order of operations is division, multiplication, subtraction, and in the end addition. This is in contrast to the tradition that reverses the natural order, which creates yet more learning problems. And that brings us to the sixth missing link, that operations, and especially addition, can be both soft and hard. This link is missing both in primary school and in secondary school.

Bo: OK, Allan. So next time we will hear about that. Can you shortly mention what the other missing links are?

Allan: Yes Bo. The seventh and eighth missing links are reversed calculations and per-numbers.

Screen 19

Bo: Thank you, Allan. This time we have heard about five missing links: The Total, from sticks to icons, cup-writing, recounting and predicting calculations. Next time on the MATHeCADEMY.net channel we will look at deconstruction of secondary mathematics and hear about the three remaining missing links.

Screen 20

Session II: Secondary school, http://youtu.be/MItYFL-3JnU

Screen 21 & 22

Bo: Welcome to the MATHeCADEMY.net Channel. And welcome to our series on deconstruction in mathematics education. My name is Bo. Today we address the question "Mathematics IS hard - or is it?" Last time we looked at primary school. This time we look at secondary school. And welcome to our guest, Allan, who uses deconstruction in his work.

Allan: Thank you Bo.

Bo: Allan, can you please repeat what deconstruction means to you?

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Bo: And what do you mean by a missing link?

Screen 23

Allan: I use the term missing link to indicate that mathematics has lost its link to its root, the natural fact Many. However, if this link is established again, then mathematics will once more become a science about Many. And accepting it as a natural science will make mathematics easy and accessible to all, instead of difficult and accessible only to few.

Bo: Will you please repeat the five missing links in primary school?

Screen 24

Allan: I will be glad to. The first missing link is the total: We only write numbers and always exclude the total. The second missing link is the missing transition from sticks to icons, showing that there are five sticks in the five-icon, etc. The third missing link is not using cup-writing to report the result of counting Many by bundling, and placing the bundles and the unbundled in separate cups, thus separating them with a decimal point and including the bundle-size as a unit. The fourth missing link is the missing use of re-counting when changing units. And the fifth missing link is the missing emphasis on mathematics as a number-predicting language.

Bo: Thank you, Allan. And that brings us to the remaining three missing links?

Screen 25

Allan: Yes Bo. The sixth missing link is the difference between soft and hard operations. Let us write the total 456 as we say it, i.e., as four hundreds, five tens and six ones. We see that we count by bundling in tens; and that we total by uniting bundles of bundles, bundles and unbundled. Thus the total is not a number but a calculation that adds three stacks: a stack of four ten-bundles of ten-bundles, and a stack of five ten-bundles and a stack of six unbundled ones. However, the three stacks are not added on-top of each other; they are added next-to each other. So in what we can call 'hard algebraic addition' stacks are added on-top of each other, while in what we can call 'soft geometric addition' stacks are added next-to each other. And adding next-to each other as geometrical stacks is in fact integration. Also we see that all numbers carry units: ones, tens, and tens-tens, also called hundreds. And finally we see the four ways we unite numbers: we add, we multiply, we power, and we integrate.

Bo: I now see why we use the Arabic word for uniting, algebra. But Allan, can you please specify a little?

Screen 26

Allan: Certainly, Bo. If we want to add one fourth and two threes we can do it in two different ways. We can add them on-top as 'hard algebraic addition'. Then the units must be the same, so we recount the 2 3s to 1.2 4s. Thus the total is 2.2 4s. Or, we can add them next-to each other as 1.3 7s.

Bo: But Allan, four times five is still twenty, isn't it?

Screen 27 & 28 & 29

Allan: Again, Bo, looking at the number 456 we see that five times ten means five tens. Likewise, four times five just means four fives, which may be, but doesn't have to be, recounted in 6s as 3.2 6s, etc., or in tens as 2.0 tens. So 'soft geometric multiplication' repeats adding on-top, while 'hard algebraic multiplication' recounts in tens, making 2 times 3 sevens not 6 sevens, but 4 tens and 2 ones = 4.2 tens = 42.

Bo: And what is the seventh missing link?

Screen 30

Allan: The seventh missing link is reversed calculation. Going forward we ask: two plus three gives what? Going backwards we ask: two plus what gives five? Instead of trying out alternatives we can predict the result by inventing an inverse operation to addition, called subtraction or minus. This allows the answer to be predicted by the calculation five minus two. So by definition five minus two is the number x that added to two gives five.

Bo: But isn't 2 + x = 5 an equation?

Screen 31

Allan: Well, an equation is just another word for reversed calculation. And since the equation 2 + x gives 5 is solved by the number x equal 5 minus 2, we see that solving an equation means isolating the unknown by moving numbers to the opposite side with the opposite sign.

Bo: Does this also apply to the other operations?

Screen 32

Allan: Indeed it does. Since 6/2 is defined as the number x that multiplied with two gives six, we see that the equation x*2= 6 again is solved by moving the number two to the opposite side with opposite sign, i.e., by x = 6/2. Likewise, the equation x to the power of 3 gives 8 is solved by moving the exponent 3 to the opposite side as the third root. And the equation 2 to which power will give eight is solved by moving the base 2 to the opposite side as the 2-based logarithm. Finally the last of the four basic operations, integration, is moved to the opposite side as its opposite operation, differentiation. Again we see that calculations predict numbers.

Bo: So what is the eighth missing link?

Screen 33 & 34

Allan: The last missing link is per-numbers. Per-numbers only occur as percentages meaning per hundred. However, recounting also gives meaning to per five, per seven, etc. So returning from sticks and strokes to the real world, all totals have physical units: meters, kilograms, hours, dollars etc. As an example we can look at coffee where 5 kg might be recounted as 8 $.

This means that the quantity-price relation can be described by the per-number 8 $ per 5 kg, or 8 per 5 $ per kg. To find the cost of 20 kg we juts recount 20 in 5s by saying T = 20 kg = (20/5) times 5 kg = (20/5) times 8$, which is 32 $.

Bo: So proportionality is basically about per-numbers?

Screen 35 & 36

Allan: Precisely! And so is integration. Integration just means adding per-numbers. To add 2kg at 3$/kg and 4 kg at 5$/kg we can add the unit-numbers to 6 kg. But we cannot directly add the per-numbers. To add the per-numbers, we transform them to unit-numbers by multiplication. This gives the area under the per-number graph. Thus the 6 kg cost 2 times 3$ plus 4 times 5$, which is 26$. So the total per-number is 26$ per 6kg. This shows that per-numbers are added by their areas, i.e. by integration, i.e. by adding next-to each other.

Bo: So Allan, you think reopening these eight links will change mathematics from hard to easy?

Screen: MANY: Count & Add; Count: decimal-numbers with units; Add on-top: Recount to shift unit; Add next-to: Integration; Add per-numbers by areas; Reversed Calculation = Equation; Move to opposite side with opposite sign.

Screen 37

Allan: Yes, Bo, I do. Mathematics becomes easy when presented as a natural science grounded in the natural fact Many. To deal with Many, we simply count and add. First we total Many by counting bundles and unbundled, using cup-writing to report counting results as decimal numbers with units. Once counted, totals can be added on-top or next-to. If added on-top, the units must be the same. So a total might have to be recounted to shift its unit. Thus recounting is the root of proportionality. And adding next-to is the root of integration, also occurring when adding per-numbers that come from recounting totals in different physical units. Finally, any calculation can be reversed, and the unknown number can be predicted by opposite operations. So to solve an equation you simply move numbers to the opposite side with opposite sign.

Bo: Allan, it seems so simple?

Allan: And Bo, it is simple. However, if you remove the eight links to its root, the natural fact Many, mathematics is transformed from a natural science into what might be called mandarin mathematics. That is why the MATHeCADEMY.net is created so teachers can learn about and teach mathematics as a natural science about Many.

Bo: What do you mean by mandarin mathematics?

Screen 38

Allan: It is interesting to compare the Korean and the Chinese alphabet. The Korean was made to allow as many as possible to read and write; and the Chinese had the opposite goal. It was created by a mandarin class possessing public jobs that they wanted their children to inherit. Consequently the mandarins invented an alphabet so difficult that only mandarin children had the time and support to learn it before the exams. The French thinker Bourdieu says that Europe might have get rid of its autocratic kings but their administrations still remain, and they still manned by a mandarin class, that use mathematics as what he calls symbolic violence to ensure that the mandarin children will inherit the public offices.

Bo: But why is mathematics special useful for this purpose?

Allan: The word language is learned when growing up, in contrast to the number-language. So by removing the eight links to its root, the natural fact Many, the mandarins have succeeded in reserving mathematics for their children. Other means are forced classes with variable time tables to prevent daily lessons from providing a serious learning.

Bo: Do you know of any mandarin class that deliberately tries to make mathematics difficult?

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Allan: There doesn't have to be one as demonstrated by the French thinker, Foucault. Foucault shows how once a certain way of talking about things, a discourse, has established itself as a discipline, it will discipline not only itself, but also its subject. Meaning, that once mandarin mathematics with its eight missing links has been established as the dominating discourse, it is impossible to talk or teach outside this discourse. And to stay in office, mandarins must follow orders given by the discourse. And in this way they reproduce the discourse. Also the subject of mathematics, the total, is forced to silence both itself and its nature as a decimal number with a unit. Instead the discourse declares that unnatural numbers without a unit and with a misplaced decimal point should be called natural. This creates huge learning problems. But it serves well the hidden agenda of the mandarins in the public offices.

Screen 40 & 41

Bo: Thank you, Allan, for sharing with us your view on the question 'mathematics IS hard – or is it?" In these two sessions we heard about the eight missing links of mandarin mathematics. Next time on the MATHeCADEMY.net channel we will look at fractions. Again we will ask: 'Fractions is hard – or is it?'