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SAVING DROPOUT RYAN WITH A TI-82

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To lower the dropout rate in pre-calculus classes, a headmaster accepted buying the cheap TI-82 for a class even if the teachers said students weren't even able to use a TI-30. A compendium called 'Formula Predict' replaced the textbook. A formula's left and right hand side were put on the y-list as Y1 and Y2 and equations were solved by 'solve Y1-Y2 = 0'. Experiencing meaning and success in a math class, the learners put up a speed that allowed including the core of calculus and nine projects.
Keywords: dropout, metamatism, model, calculator, regression, formula, equation, per-number

THE TASK: REDUCE THE DROPUT RATE!

The headmaster asked the mathematics teachers: "We have too many pre-calculus dropouts. What can we do?" I proposed buying the cheap TI-82 graphical calculator, but the other teachers rejected this proposal arguing that students weren't even able to use a simple TI-30. Still I was allowed to buy this calculator for my class allowing me to replace the textbook with a compendium emphasizing modeling with TI-82.

CONCEPTS: EXAMPLES OF ABSTRACTIONS OR VICE VERSA

Enlightenment mathematics was as a natural science exploring the natural fact Many (Kline, 1972) by grounding its abstract concepts in examples, and by using the lack of falsifying examples to validate its theory. But after abstracting the set-concept, mathematics was turned upside down to modern mathematics or 'metamatism', a mixture of 'meta-matics' defining its concepts as examples of abstractions, and 'mathematism' true in the library, but not in the laboratory, as e.g. 2+3 = 5, which has countless counterexamples: 2m+3cm = 203 cm, 2weeks+3days = 17 days etc. Being self-referring, this modern mathematics did not need an outside world. However, a self-referring mathematics turned out to be a self-contradiction. With his paradox on the set of sets not belonging to itself, Russell proved that sets implies self-reference and self-contradiction as known from the classical liar-paradox 'this statement is false' being false when true and true when false: If $M = \{A \mid A \notin A\}$, then $M \in M \Leftrightarrow M \notin M$. Likewise Gödel proved that a well-proven theory is a dream since it will always contain statements that can be neither proved nor disproved.

In spite of being neither well-defined nor well-proven, mathematics still teaches metamatism. This creates big problems to mathematics education as shown e.g. by 'the fraction paradox' where the teacher insists that 1/2 + 2/3 IS 7/6 even if the students protest: counting cokes, 1/2 of 2 bottles and 2/3 of 3 bottles gives 3/5 of 5 as cokes and never 7 cokes of 6 bottles.

CONTINGENCY RESEARCH UNMASKS CHOICES PRESENTED AS NATURE

Alternatively, mathematics could return to its roots, Many, guided by contingency research uncovering hidden patronization by discovering alternatives to choices presented as nature.

Ancient Greece saw a controversy on democracy between two different attitudes to knowledge represented by the sophists and the philosophers. The sophists warned that to practice democracy, the people must be enlightened to tell choice from nature in order to prevent hidden patronization by choices presented as nature. To the philosophers, patronization was a natural order since to them all physical is examples of meta-physical forms only visible to the philosophers educated at Plato's academy, who therefore should be given the role as natural patronizing rulers (Russell, 1945).

Later Newton saw that a falling apple obeys, not the unpredictable will of a meta-physical patronizor, but its own predictable physical will. This created the Enlightenment: when an apple obeys its own will, people could do the same and replace patronization with democracy.

Two democracies were installed: one in the US still having its first republic; and one in France, now having its fifth republic. German autocracy tried to stop the French democracy by sending in an army. However, a German mercenary was no match to a French conscript aware of the feudal consequence of defeat. So the French stopped the Germans and later occupied Germany. Unable to use the army, the German autocracy instead used the school to stop enlightenment in spreading from France. As counter-enlightenment, Humboldt used Hegel philosophy to create a patronizing line-organized Bildung school system based upon three principles: To avoid democracy, the people must not be enlightened; instead romanticism should install nationalism so the people sees itself as a 'nation' willing to fight other 'nations', especially the democratic ones; and the population elite should be extracted and receive 'Bildung' to become a knowledge-nobility for a new strong central administration replacing the former blood-nobility unable to stop the French democracy.

As democracies, EU still holds on to line-organized education instead of changing to block-organized education as in the North American republics allowing young students to uncover and develop their personal talent through individually chosen half-year knowledge blocks.

In France, the sophist warning against hidden patronization is kept alive in the post-structural thinking of Derrida, Lyotard, Foucault and Bourdieu. Derrida warns against ungrounded words installing what they label, such word should be 'deconstructed' into labels. Lyotard warns against ungrounded sentences installing political instead of natural correctness. Foucault warns against institutionalized disciplines claiming to express knowledge about humans; instead they install order by disciplining both themselves and their subject. And Bourdieu warns against using education as symbolic violence to monopolize the kno wledge capital for a knowledge-nobility (Tarp, 2004).

Thus contingency research does not refer to, but questions existing research by asking 'Is this nature or choice presented as nature?' To prevent patronization, categories should be grounded in nature using Grounded Theory (Glaser et al, 1967), the method of natural research developed in the first Enlightenment democracy, the American, and resonating with Piaget's principles of natural learning (Piaget, 1970).

THE CASE OF TEACHING MATH DROPOUTS

Being our language about quantities, mathematics is a core part of education in both primary and secondary education. Most parents accept the importance of learning mathematics, but many students fail to see the meaning in doing so. Consequently special core math courses for dropouts are developed.

TRADITIONS OF CORE PRECALCULUS COURSES FOR DROPOUTS

A typical core course for math dropouts is halving the content and doubling the text volume. So in a slow pace the students work their way through a textbook once more presenting mathematics as a subject about numbers, operations, equations and functions applied to space, time, mass and money. To prevent spending time on basic arithmetic, a TI-30 calculator is handed out without instruction.

As to numbers, the tradition focuses on fractions and how to add fractions.

Then solving equations is introduced using the traditional balancing method isolating the unknown by performing identical operations to both sides of the equation. Typically, the unknown occur on both sides of the equation as $2x + 3 = 4x – 5$; or in fractions as $5 = 40/x$.

Then relations between variables are introduced using tables, graphs and functions with emphasis on the linear function $y = f(x) = b+a*x$. In a traditional curriculum, a linear function is followed by the quadratic function. But a core course might instead go on to the exponential function $y = b*a^x$. To avoid solving its equations the solutions are given as formulas.

PROBLEMS IN TRADITIONAL CORE COURSES

The intention of a traditional core course is to give a second chance to learners having dropped out op the traditional math course. However, from a skeptical viewpoint trying to avoid hidden patronization by presenting choice as nature, several questions can be raised.

As to numbers, are fractions numbers or calculations that can be expressed with as many decimals as we want, typical asking for three significant figures? Is it meaningful to add fractions without units as shown by the fraction-paradox above?

As to equations, is the balancing method nature or choice presented as nature? The number $x = 8-3$ is defined as the number x that added with 3 gives 8, $x + 3 = 8$. This can be restated as saying that the equation $x + 3 = 8$ has the solution $x = 8-3$, suggesting that the natural way to solve equations is the 'move to opposite side with opposite sign' method. This method can be applied to all cases of reversed calculation:

$x + 3 = 15$	$x*3 = 15$	$x^3 = 125$	$3^x = 243$
$x = 15 – 3$	$x = 15/3$	$x = 3\sqrt{125}$	$x = \log3(243)$

Figure 1. Equations solved by the 'opposite side & sign' method of inverse calculation

Defining a function as an example of a set-relation, is that nature or choice presented as nature? In a formula as $y = a+b$ all numbers might be known. If one number is unknown we have an equation to be solved, e.g. $5 = 3+x$, if not already solved, $x = 3+5$. With two unknown numbers we have a function as in y = 3+x, or a relation as in 3 = x+y that can be changed into the function y = 3–x. So a function is just a label for a formula with two unknown variables.

Giving solution formulas to the exponential equations x^3 = 125 and 3^x = 243, is that nature or choice presented as nature? Since equation is just another name for inverse calculation, using the inverse operations root and log is the natural way to solve exponential equations.

The prime goal of education is that learners adapt to the outside world by proper actions. An action as 'Peter eats apples' is a three-term sentence with a subject, a verb and an object. Thus mathematics education should be described in this way. The learner is the subject, the object is the natural fact Many, and the verb is how we deal with Many: we totalize expressing the total as a formula, e.g. T = 345 = 3*10^2 + 4*10 + 5*1 showing that totalizing means counting and adding bundles, that all numbers carry units, and that there are four ways to add: +, *, ^ and integration adding unlike and like unit-numbers and like and unlike per-numbers.

Totalizing can also be called algebra if using the Arabic word for reuniting. Not being a verb, mathematics could be renamed to 'totalizing', 'counting and adding' or reckoning.

DESIGING A GROUNDED MATH CORE COURSE

Real word problems translate to formulas by modeling, or triangulation in the case of forms. So, to adapt to the outside world, mathematics education has as its prime goal that persons learn how to totalize, i.e. how to count and add, how to model and how to triangulate.

A traditional core course seems to be filled with examples of choices presented as nature. This leads to the question: is it possible to design an alternative core course based upon nature instead of choices presented as nature? In other words, what would be the content of a core course in pre-calculus if grounded in the roots of mathematics, the natural fact Many?

Mathematics as a Number-language using Predicting Formulas

As to the nature of the subject itself, mathematics is a number-language that together with the word-language allows users to describe quantities and qualities in everyday life. Thus a calculator is a typewriter using numbers instead of letters. A typewriter combines letters to words and sentences. A calculator combines digits to numbers that combined with operations become formulas. Thus formulas are the sentences of the number-language.

A difference between the word-language and the number-language is that sentences describe whereas formulas predict the four different ways of uniting numbers:

Addition predicts the result of uniting unlike unit-numbers: uniting 4$ and 5$ gives a total that is predicted by the formula T = a+b = 4+5 = 9

Multiplication predicts the result of uniting like unit-numbers: uniting 4$ 5 times gives a total that is predicted by the formula T = a*b = 4*5 = 20.

Power predicts the result of uniting like per-numbers: uniting 4% 5 times gives a total that is predicted by the formula 1+T = a^b = 1.04^5 = 1.217, i.e. T = 0.217 = 21.7%.

Integration predicts the result of uniting unlike per-numbers: uniting 2kg at 7$/kg and 3kg at 8$/kg gives 5 kg at T$/5kg where T is the area under the $/kg per-number graph, T = Σ p*Δx.

Solving Equations with a Solver

As shown above, inverse operations solve equations, as do the TI-82 using a solver. Equations as 2+x = 6 always has a left hand side and a right hand side that can be entered on the calculators Y-list as Y1 and Y2. So any equation has the form Y1 = Y2, or Y1 – Y2 = 0 that only has to be entered to the solver once. After that, solving equations just means entering its two sides as Y1 and Y2. Using graphs, Y1 and Y2 becomes two curves having the same values at intersection points.

If one of the numbers in a calculation is unknown, then so is the result. A table describes a formula with two unknowns by answering the question 'if x is this, then what is y?' Graphing a table allows the inverse question to be addressed by reading from the y-axis.

Producing Formulas with Regression

Once a formula is known, it produces answers by being solved or graphed. Real world data often come as tables, so to model real world problems we need to be able to set up formulas from tables. Simple formulas describe levels as e.g. cost = price*volume. Calculus formulas describe predictable change where pre-calculus describes constant change.

If a variable y begins with the value b and x times changes by a number a, then y = b+a*x. This is called linear change and occurs in everyday trade and in interest-free saving.

If y begins with the value b and x times changes by a percentage r, then y = b*(1+r)^x = b*a^x since adding 5% means multiplying with 105% = 100%+5%= 1.05. This is called exponential change and occurs when saving money in a bank and when populations grow or decay.

In the case of linear and exponential change, a two-line table allows regression on a TI-82 to find the two constants b and a.

Multi-line tables can be modeled with polynomials, the formula used to describe numbers. Thus a 3-line table might be modeled by a degree 2 polynomial y = b + a*x + c*x^2 including also a bending-number c; and a 4-line table by a degree 3 polynomial y = b + a*x + c*x^2 + d*x^3 including also a counter-bending number d, etc.

Graphically, a degree 2 polynomial is a bending line, a parabola; and a degree 3 polynomial is a double parabola. The top and the bottom of a bending line as well as its zeros can be found directly by graphical methods of the TI-82.

Fractions as Per-numbers

Fractions are rooted in per-numbers: 3$ per 5 kg = 3$/5kg = 3/5 $/kg. To add per-numbers they first must be changed to unit numbers by being multiplied with their units:

3 kg at 4 $/kg + 5 kg at 6 $/kg = 8 kg at (3*4 + 5*6)/8 $/kg

Geometrically this means that the areas under their graph add per-numbers.

Her again TI-82 comes in handy when calculating areas under graphs. Areas can be calculated also in the case where the graph is not horizontal but a bending line, representing the case when the per-number is changing continuously as e.g. in a falling body: 3 seconds at 4 m/s increasing to 6 m/s totals 15 m because of a constant acceleration.

Models as Quantitative Literature

Using TI-82 as a quantitative typewriter able to set up formulas from tables and to answer both x- and y-questions, it becomes possible to include models as quantitative literature.

All models share the same structure: A real world problem is translated into a mathematical problem that is solved and translated back into a real world solution to be evaluated.

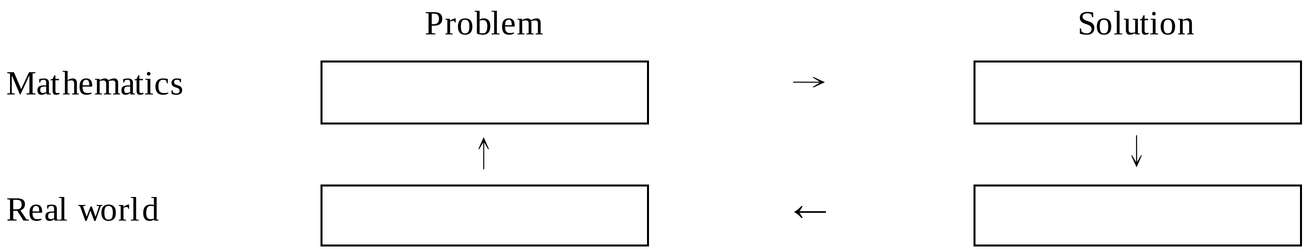


Figure 2. The four phases of mathematical modeling

The project 'Population versus food' looks at the Malthusian warning: If population changes in a linear way, and food changes in an exponential way, hunger will eventually occur. The model assumes that the world population in millions changes from 1590 in 1900 to 5300 in 1990 and that food measured in million daily rations changes from 1800 to 4500 in the same period. From this 2-line table regression can produce two formulas: with x counting years after 1850, the population is modeled by Y1= 815*1.013^x and the food by Y2= 300 + 30x. This model predicts hunger to occur 123 years after 1850, i.e. from 1973.

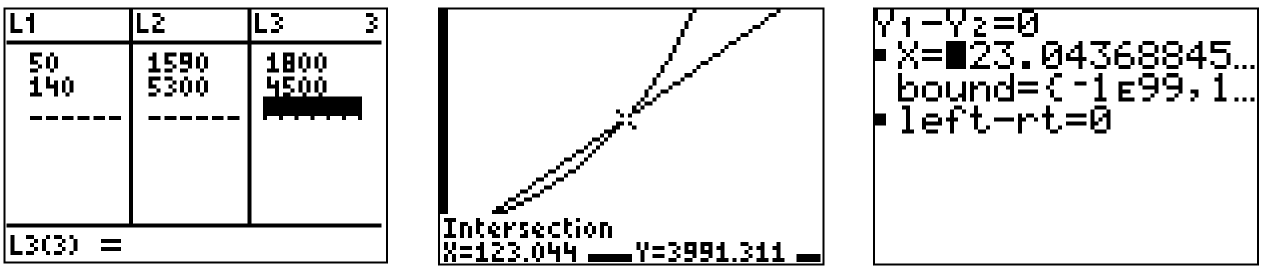


Figure 3. A Malthusian model of population and food levels

The project 'Fundraising' finds the revenue of a fundraising assuming all students will accept a free ticket, that 100 students will buy a 20$ ticket and that no one will buy a 40$ ticket. From this 3-line table the demand is modeled by a degree 2 polynomial Y1 = .375*x^2 – 27.5*x + 500. Thus the revenue formula is the product of the price and the demand, Y2 = x*Y1. Graphical methods shows that the maximum revenue will be 2688 $ at a ticket price of 12$.

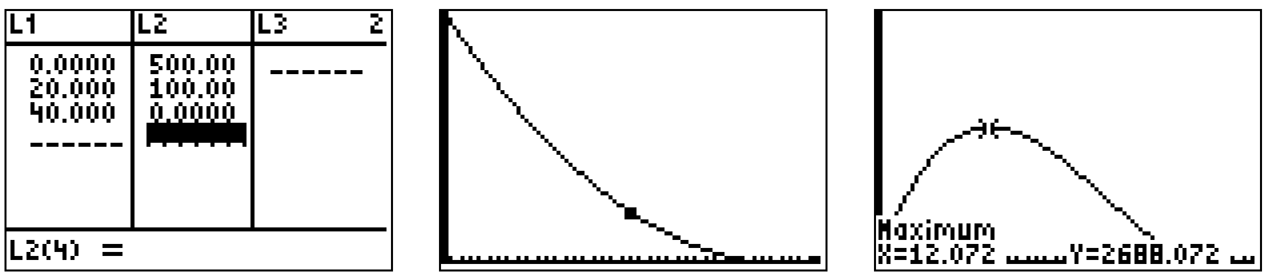


Figure 4. Modeling the optimal ticket price in a fundraising

In the project ‘Driving with Peter’ his velocity is measured five times. A model can answer many questions, e.g. when was Peter accelerating? And what distance did Peter travel in a given time interval? From a 5-line table the speed can be modeled by the degree 4 polynomial $Y1 = -0.009x^4 + 0.53x^3 - 10.875x^2 + 91.25x - 235$. Visually, the triple parabola fits the data points. Graphical methods shows that a minimum speed is attained after 14.2 seconds; and that Peter traveled 115 meters from the 10th to the 15th second.

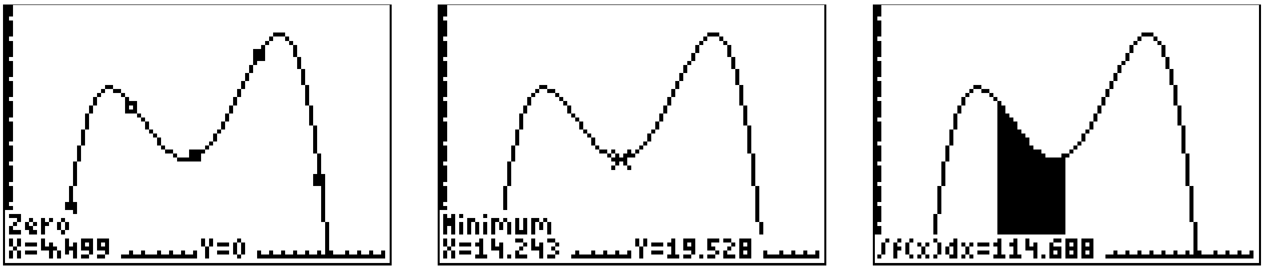


Figure 5. Modeling a car in motion

Six other projects were included in the course. The project ‘Forecasts’ modeled a capital growing constantly in three different ways: linear, exponential and potential. The project ‘Determining a Distance’ uses trigonometry to predict the distance to an inaccessible point on the other side of a river. The project ‘The Bridge’ uses trigonometry to predict the dimensions of a simple expansion bridge over a canyon. The project ‘Playing Golf’ predicts the formula for the orbit of a ball that has to pass three given points: a staring point, an ending point and the top of a hedge. The project ‘Saving and Pension’ predicts the size of a ten years monthly pension created by a thirty years monthly payment of 1000$ at an interest rate of 0.4% per month. And the project ‘The Takeover Attempt’ predicts how much company A has spent buying stocks in company B given an oscillating course described by a 4-line table.

INTRODUCING THE THREE GENRES OF QUANTITATIVE LITTERATURE

Qualitative and quantitative literature has three genres: fact, fiction and fiddle (Tarp, 2001).

Fact models quantify and predict predictable quantities, as e.g. ‘What is the area of the walls in this room?’ In this case the calculated answer of the model is what is observed. Hence calculated values from a fact models can be trusted. A fact model can also be called a since-then model or a room-model. Most models from basic science and economy are fact models.

Fiction models quantify and predict unpredictable quantities, as e.g. ‘My debt will soon be paid off at this rate!’ Fiction models are based upon assumptions and its solutions should be supplemented with predictions based upon alternative assumptions or scenarios. A fiction model can also be called an if-then model or a rate-model. Models from basic economy assuming variables to be constant or predictable by a linear formula are fiction models.

Fiddle models quantify qualities: ‘Is the risk of this road high enough to cost a bridge?’ Many risk-models are fiddle models. The basic risk model says: Risk = Consequence * probability. Statistics can provide probabilities for casualties, but if casualties are quantified, it is much cheaper to stay in a cemetery than in a hospital, pointing to the solution ‘no bridge’. Fiddle models should be rejected asking for a word description instead of a number description.

INTRODUCING THE CORE OF CALCULUS AS ADDING PER-NUMBERS

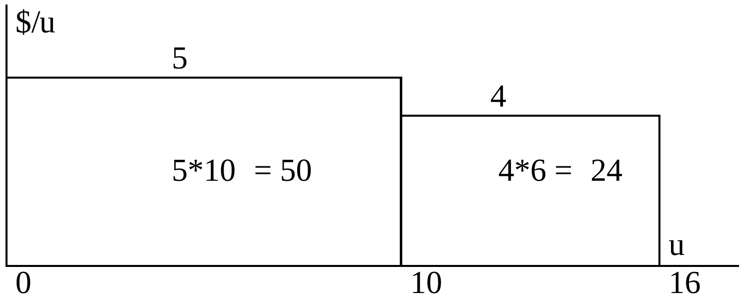
As an introduction to calculus the students looked at discounts: The price 5 $/u goes down to 4 $/u when buying more than 10 units, what is the price when buying 16 units?

10 units at 5 $/u gives 10*5 = 50 $

6 units at 4 $/u gives 6*4 = 24 $

16 units at 9 $/u gives 16*9 = 74 $

?? How do we add per-numbers??



The problem is that where unit-numbers are added directly, per-numbers are added as areas under the per-number graph, i.e. as Σ p*Δx, written by TI.82 as $\int$ p dx.

So if a per-number p is constant, the total cost T for buying 5 units is T = p*5. And if the per-number p is a formula, the total cost T for buying 5 units is the area under the p-graph.

COMPLETING THE ALGEBRA PROJECT

Seeing the use of integration as adding per-numbers, the students enjoyed having completed the reuniting project of algebra since now they were able to add all four number types:

	Unlike	Like
Unit-numbers *m, s, kg, $*	**T = a + b** *T – b = a*	**T = a*b** *T/b = a*
Per-numbers *m/s, $/kg, $/(100$) = %*	**T = $\int$ p dx** *dT/dx = p*	**T = a^b** *b√T = a* *loga(T) = b*

STUDENTS ASKING FOR PROOFS

To stop fellow students mocking them by saying that the class was not on mathematics but on reckoning, the students asked for sophisticated proofs. Four were sufficient.

Depositing n times the interest r*(a/r) = a$ of a/r $ to an account makes this a saving account containing the total interest A = R*(a/r). Consequently A/a = R/r where 1+R = (1+r)^n.

In a triangle ABC with no angle above 90, the outside squares of the sides are divided in rectangles by the heights. Projections show that the two rectangles containing C have the same area, a*b*cosC. This gives c^2 = a^2 + b^2 – 2*a*b*cosC, or c^2 = a^2 + b^2 if C is 90.

Adding 100% interest in n parts n times gives the growth multiplier m = (1+1/n)^n. For n sufficiently big, m = 2.7182818 = e. Thus compounding 100% can add at most 71.8%.

Inscribing a symmetrical n-radius star in a unit circle gives n intersection points, from which tangents create a polygon with circumference c = 2*n*tan(180/n). For n big, c gets close to that of the unit-circle, 2*π. Hence π = n*tan(180/n) = 3.14159 for n sufficiently big.

TESTING THE CORE COURSE

The students expressed surprise and content with the course. Their hand-ins were on time. And the course finished before time giving room for additional models from classical physics: vertical falling balls, projectile orbits, colliding balls, circular motion, pendulums, gravity points, drying wasted whisky with ice cubes and supplying bulbs with energy.

At the written and the oral exam, for the first time at the school, all the students passed. Some students wanted to move on to a calculus class, other were reluctant arguing that they had already learned the core of calculus so they didn't need an extra class to study engineering.

REPORTING BACK TO THE HEADMASTER

The headmaster expressed satisfaction, but the teachers didn't like setting aside the textbook and its traditional mathematics. To encourage the teachers, the headmaster ordered the TI-82 to be bought for all pre-calculus classes.

DISCUSSING THE RESULT

Discussing the result with teachers, an argument was that by not following the textbook, the students don't learn mathematics, only reckoning. My answer was that in the first place, the textbook does not teach mathematics but metamatism, so what dropouts reject is metamatism, not mathematics. Furthermore, teachers should respect the Nuremberg sentences from 1946: You can't just follow orders, you must evaluate the consequenses to your clients. Your order is to teach mathematics, but it must be useful and not harmful to the learners. So skepticism should reformulate the orders, e.g. to: the goal of mathematics education is to adapt learners to the natural fact Many by totalizing developing basic competences in how to count, add, model and triangulate; and how to cooperate with calculation technology able to perform both forward and backward calculation and to illustrate calculations with tables and graphs. Another argument was that by using technology, the students would not understand mathematics. My answer was that world problems can't be translated into formulas without understanding how the different operations are defined and used for forward and backward calculation. However, just as multiplication is useful to speed up addition, a graphical calculator should be allowed to speed up modeling so the human brain can be used to formulate questions and evaluate answers.

Discussing the result with researchers, a typical argument was that this work does not build on traditional theory in mathematics education research, e.g. theory about concept formation. My answer was that in most cases research is describing education in metamatism, not in mathematics. And in many cases research produces political instead of natural correctness as shown by the 'pencil paradox': Placed between a ruler and a dictionary, a '17 cm long pencil' can point to '15', but not to 'knife', so being itself able to falsify its number but not its word means that numbers and words produce natural and political correctness respectively. Only contingency research can produce natural correctness by uncovering hidden alternatives to choices presented as nature. Consequently, contingency research is very effective as action research assisting an actor in a field to implement change as in this case. But contingency research is banned from discourse protecting EU universities as predicted by Lyotard (1984).

CONCLUSION: HOW TO MAKE LOSERS USERS

In order to give dropout students in mathematics an extra chance it has good meaning to create a course boiling the mathematics content down to its core. However, to be successful, the core should be grounded in its roots, the natural fact Many. Numbers should be presented as polynomials to show the four operations uniting numbers according to Algebra's reuniting project. Also direct and inverse operations should be presented as means to predict the united total and its parts. In this way the core of basic algebra becomes solving equations with the move and change sign method, or with the solver of a graphical calculator. And the core of pre-calculus becomes regression, enabling tables to be translated to formulas that can be processed when entered into the y-list of the TI82. Thus grounding mathematics in its natural roots and including a graphical calculator provides ordinary students with a typewriter that can be used to model and predict the behavior of real world quantities (Tarp 2009). In this way a TI-82 develops competences in human-technology cooperation: Humans get the data and the questions, and technology provides the answers. Traditional metamatism makes losers of Ryan and other boys. Replacing metamatism with grounded mathematics and a TI-82 will not only save them, it will also install in them a confidence and a belief that they can become successful engineers cooperating with technology instead of being math dropouts. Thus learning technology based grounded mathematics in an educational system that is transformed from being line-organized to block-organized will transform boys from being dropouts to engineers, which again will help the EU solve its present economical crisis.

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