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# POSTMODERN SKEPTICISM TOWARD MATHEMATICS AND EDUCATION AND RESEARCH

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*The cornerstones of modern society is research and education, especially in the two basic languages, the word-language assigning words and sentences to qualities, and the number-language assigning numbers and calculations to quantities. And as an important institution, mathematics education is equipped with its own research to make it successful. Still the problems in mathematics education seem to grow with the number of research articles. This irrelevance paradox makes postmodern skepticism ask: Are mathematics, education and research what they claim to be? Or are they choices that presented as nature install patronization to be unmasked by postmodern contingency research? Keywords: Skepticism, postmodern, patronization, metamatism, totalize, reckon*

## THE BACKGROUND

The modern world began when natural science replaced belief with certainty by using the outside world to provide reliable data, and to falsify deduced predictions to test their validity. This enlightening method created the Enlightenment century wanting to replace autocratic patronization with democratic voting. Hence schools were created as enlightenment institutions with special focus on the three basic Rs: how to Read, how to wRite and how to Reckon. However, today reckoning enlightenment is called mathematics education supplied with its own research and facing huge learning problems as e.g. formulated in 'the relevance paradox formed by the simultaneous objective relevance and subjective irrelevance of mathematics' (Niss in Biehler et al, 1994, p. 371). What went wrong? Let us ask research.

## SCRIPTURES ROOT PATRONIZING RESEARCH

In ancient Greece the Pythagoreans formed a closed society to discover metaphysical laws in the physical world. They found three. Halving the length of a vibrating string makes the tone go up one octave; and other proportions also create harmony. In a triangle, two angles can be chosen freely, but a metaphysical law binds the third. Likewise with the sides in a right angled triangle. From their discoveries the Pythagoreans formulated the thesis 'all is numbers'. Plato generalized this by saying 'all is forms', meaning that physical phenomena are examples of metaphysical forms only visible to the philosophers educated at the Plato academy, and who consequently should be accepted as social patronizors (Russell, 1945).

The Christian church took over the idea of metaphysical patronization, but changed the academies to monasteries with corridors housing cells with writers commenting on the Holy Scriptures, and calling their interpretations research using a hermeneutic method.

Using a physical method to discover natural correctness, natural science created the modern university taking over the monastery. However, hermeneutics stayed so today two kinds of descriptions claim to be scientific, one using numbers and the other using words; in spite of the fact that placed between a ruler and a dictionary, a stick can point to 15 but not to 'pencil'.

To postmodern skepticism, natural sciences produce enlightening natural correctness, while social and human sciences produce patronizing political correctness.

## SKEPTICISM ROOTS POSTMODERN CONTINGENCY RESEARCH

Skepticism towards hidden patronization is the root of postmodern thinking as formulated in Lyotard's definition 'Simplifying to the extreme, I define postmodern as incredulity toward metanarratives (Lyotard, 1984: xxiv).'

In the first republic, the ancient Greek sophists expressed skepticism: to prevent patronization by choices presented as nature, the people must be enlightened to tell choice from nature, i.e. to tell political correctness from natural. So authority should come from voting instead of 'Gewalt', claiming that by nature ordinary people, like children, do not possess 'Mündigkeit'; a continental concept that the lack of autocracy never made relevant in the English language.

In the Renaissance, Brahe grounded his skepticism in physical observations allowing Newton to replace a political correct geocentric theory with a natural correct heliocentric theory.

Discovering physical will, Newton inspired French Enlightenment thinking: if falling apples obey their own will, then people could do the same and replace patronization with voting. Two republics were formed, an American and a French.

The US still has its first republic using pragmatism and grounded theory to exert skepticism towards philosophical patronization. The French has its fifth republic, repeatedly turned over by German neighbors, which has created a French skeptical poststructuralist thinking that warns against patronization hidden in words, sentences, institutions and education.

Derrida thus uses the term 'deconstruction' to warn against patronizing words installing instead of labeling what they name. Lyotard uses the term 'postmodern' to warn against patronizing sentences stating political instead of natural correctness. Foucault uses the term 'pastoral power' to warn against the human disciplines using self-reference to discipline themselves and their subject by using is-statements to install diagnoses to be cured by normalizing institutions using the methods of the same human disciplines. And Bourdieu uses 'symbolic violence' to label patronizing education giving monopoly of knowledge capital to a knowledge nobility; and sees mathematics as especially useful to that end. (Tarp, 2004)

Based upon this French warning against hidden patronization, a research paradigm can be created called postmodern 'contingency research' unmasking hidden patronization by finding alternatives to choices presented as nature. Categories and discourses are non-patronizing if grounded in nature using Grounded Theory (Glaser et al, 1967), the natural research method developed in the US enlightenment democracy; and resonating with Piaget's principles of natural learning, saying that individuals adapt to the outside world by creating schemata to assimilate the outside world or to be accommodated to the outside world (Piaget, 1970); both resonating with the inductive method used by natural science to produce natural correctness.

## NATURAL AND POLITICAL ROOTED EDUCATION

To adapt to the outside world reptiles use genes, mammals also use parental care, and humans also use language, developed when motion-freed forelegs created additional brain capacity to keep the balance and to store the sounds associated with what was grasped with the forelegs, thus supplying humans with a language to share information through communication that, if institutionalized, is called schooling or education, typically containing primary education for children, secondary education for teenagers and tertiary education for adults.

In primary education, children learn basic communication skills, i.e. reading, writing and reckoning, and obtain a general knowledge about nature and society in time and space. Here it makes good sense to concentrate children of the same age in the same class.

In a republic with few institutions as the American, you earn your living from your talents uncovered and developed through daily lessons in self-chosen half-year blocks in secondary and tertiary schools. Block-organized education creates a flexible workforce that adapt quickly to changes in technology, e.g. when modernity, created by the invention of artificial muscles that combined with tools became machines to do the hard physical jobs, changed to postmodernity created by the invention of artificial brains, computers, that combined with machines become robots to perform routine information and production jobs.

In more autocratic states, primary education was sufficient background for being an unskilled or skilled worker. But, exercising its Gewalt through institutions, the state needed skilled officials with secondary and tertiary exams leading directly to office. Institutionalization thus makes line-organized office-preparing education a rational choice if only 10% of a youth population continue in secondary and tertiary education, all wanting to qualify for an office. But line-organization becomes problematic when 95% need a secondary education in the information economy of postmodernity where robots have taken over many traditional jobs.

As office preparation line-organized education teaches learners to follow orders, which may be OK in industry controlled externally by the correctness of the market, but might become problematic in institutions controlled internally by political correctness. Bauman and Arendt point out that, by following orders, modernity can create both gas turbines and gas chambers. Following authorized routines, institutions cannot know if they produce benefit or crime against humanity as shown in the Nuremberg Trials (Bauman, 1989; Arendt, 2005).

Using line-organized education to educate a whole population involves non-democratic patronization. The learners are concentrated in age-determined classes, and forced to choose its multi-year block-combination with exams in the end that cannot be retaken. The use of KZ-like techniques to practice forced classes and forced feeding prevents learners from leaving unsuccessful subjects, instead they are forced to leave education entirely, thus having their personal talents left unnourished. And the inflexibility of line-organization forces people to stay so long in education that the EU reproduction rate now is 1.5 children per family, reducing the EU population to 10% in 200 years. Although efficient in an autocracy wanting to exercise state Mündigkeit, line-organized education is a disaster in a democracy accepting adult Mündigkeit. However, EU finds it hard to learn the Nuremberg lesson: Don't exaggerate institutionalization, and don't use forced concentration for crowd-control.

## SELFREFERENCE CHANGES MATHEMATICS INTO METAMATISM

Although created as abstractions from external examples, the set-concept allowed mathematical concepts to be internally defined as examples of abstractions. But a self-referring mathematics was soon proven contradictory. Being false when true and true when false, the classical liar-paradox 'this statement is false' inspired Russell to formulate a paradox about the set of sets not belonging to itself: If $M = \{ A \mid A \notin A \}$, then $M \in M \Leftrightarrow M \notin M$ (Kline, 1972).

To avoid this paradox, a set-theory was invented not distinguishing between elements and sets, thus being meaningless by mixing examples and abstractions: you can eat an example of an apple, but not the abstraction 'apple'. Also Peano axioms were invented using a follower principle to prove that $1+1 = 2$ is a natural correct statement. However, in a laboratory, 1 week + 1 day = 8 days, and 1 is not well defined since 1 threes = 3 ones and 1 fives = 5 ones etc.

Thus two terms can be coined: In 'meta-matics' definitions are examples of abstractions instead of abstractions from examples; and 'mathe-matism' produces statements true in a library, but not in a laboratory, as e.g. '1+1 = 2'. Together, meta-matics and mathe-matism form 'meta-matism' preaching politically correct, but ungrounded concepts and statements.

## THE NATURAL FACT MANY MAKES MATHEMATICS A NATURAL SCIENCE

The natural fact Many occurs in time and space as repetition and multiplicity. To deal with Many, we totalize by counting and adding. Counted by bundling, a total T becomes a number of unbundled, bundled, bundles of bundled etc. Thus a natural number includes a unit and a decimal point to separate the bundled from the unbundled, e.g., T = 9 1s = 2.1 4s. This way of counting allows mathematics to be learned with 1digit numbers alone (Zybartas et al, 2005).

Once counted, totals can be united, called algebra using an Arabic word. United next-to, the units are integrated: 2 3s + 4 5s = 3.2 8s. United on-top, the units must be changed to be the same: 2 3s + 4 5s = 1.1 5s + 4 5s = 5.1 5s. Changing units is called proportionality. In the case of overloads, also bundles are bundled, e.g. making 5 fives to 1 five-fives: 5.1 5s = 10.1 5s.

Instead of counting on, operations are invented to predict the result by calculations. Thus 3+5 predicts the result of counting on 5 times from 3. The calculation 3*5 predicts the result of adding 3 5 times, and 3^5 predicts the result of multiplying with 3 5 times.

Uniting on-top and next-to provide four ways to add: Plus adds unlike unit-numbers: 3 cokes and 5 cokes = (3+5) cokes, multiplication adds like unit-numbers: 3 cokes 5 times = (3*5) cokes; power adds like per-numbers: 12% 5 times = 76% since $1.12^5 = 1.76$; and integration adds unlike per-numbers: 2 kg at 3 \$/kg + 4 kg at 5 \$/kg = 6 kg at (2*3+4*5)/6 \$/kg.

Inverse operations predict the result of reversing uniting. Thus the splitting 8 = 5+x is predicted by x = 8–5, the splitting 8 = 5*x is predicted by x = 8/5, the splitting 8 = 5^x is predicted by x = log5(8), and 8 = x^5 is predicted by x = 5√8. Calling reversed calculation equations, the natural way to solve equations is: move to the opposite side with opposite sign.

A formula shows how a total is calculated, e.g. T = 2+3*4. Variable numbers are unknown and written as letters, e.g. y = 2+3*x. This transforms a formula into a function, using tables and graphs (and a graphical calculator) to describe corresponding numbers (Tarp, 2009).

**CONCLUSION**

From a postmodern skeptical viewpoint, what is called mathematics may turn out to be metamatism rooting its concepts in internal abstractions instead of in external examples, and producing statements that are true in a library but not in a laboratory. And what is called education may turn out to be office preparation in line-organized forced classes adapting learners to internal political correctness, instead of talent development in self-chosen half-year blocks adapting learners to external natural correctness. And what is called research may turn out to be orthodoxy forcing the outside world to assimilate, instead of accommodating itself to the outside world. Consequently, mathematics education must choose between adapting to internal political correctness or external natural correctness. If choosing to adapt the learner to the natural fact Many, the diagnose-word mathematics must be replaced by action-words as totalize, count, add, reckon, triangulate etc. Likewise, post-primary education must be organized as daily lessons in self-chosen half-year blocks to allow learners to try different approaches to different subjects. As to research, instead of disciplining itself and its subject with political correctness, it should produce natural correctness by searching for hidden contingency uncovering alternatives to patronizing choices presented as nature.

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## POSTSCRIPT

*Manuscript to the YoyTube video on Postmodern Math Education where Paul Ernest and Allan Tarp discuss 8 questions: What is meant by postmodern? What is meant by modern? What is the root of postmodern thinking? Who is the most important postmodern thinker? What is mathematics? What is postmodern mathematics? What is postmodern research?*

*http://youtu.be/ArKY2y_ve_U*

Bo: Welcome to this discussion on Postmodern Mathematics education. My name is Bo. And welcome to our two guests. Welcome to Paul.

Paul: Thank you Bo. I am really looking forward to this discussion.

Bo: And welcome to Allan.

Allan: Thank you Bo, so am I.

Bo: I will ask you eight questions. The first question is: what is meant by postmodern?

Paul: As I see it, postmodernism means the rejection of a single all-encompassing metanarrative – whether it be Freudianism, Marxism, Logicism, Radical Constructivism, Enactivism, even Bourbakianism. Instead it means acceptance of multiple perspectives offering new ways of seeing mathematics, teachers and learners. Thus it is important to recognize that all human subjects have multiple selves and that we all (mathematicians, teachers and learners) have access to different selves: authoritative knowers, researchers, learners, appreciators and consumers of popular and other cultures, as well as having non-academic selves.

Allan: It seems to me that we must distinguish between post-modernism and post-modernity. Post-modernity is what we do with our hands, i.e. how we act in the world. And post-modernism is what we do with our head, i.e. how we think about the world. To simplify, post-modernity is the social condition that was created by I,T, information technology. And postmodernism is skepticism toward hidden patronization.

Paul: I would agree that postmodernism is a conceptual position. Rather it is several positions because we should distinguish cultural postmodernism (in art, architecture, music, design and fashion) from philosophical postmodernism. This is more about the rejection of a single all-encompassing theoretical metanarrative. I see post-modernity to be the epoch when postmodern ideas are current. But this is semantic and I accept your distinction between the theory and practice of the postmodern

Bo: The second question is: 'What is meant by modern?'

Paul: To me modern thinking began with Descartes, who puts forward a logical master plan to provide certain and indubitable foundations for all of knowledge. This begins with a small basis of clear and distinct ideas, and then deduces all subsequent truths by the clear rules of logic. This plan was modeled on the axiomatic geometry of Euclid, already two thousand years old, which Hobbes called the only true science bestowed on humankind.

Allan: To me, modernity means the social condition created by the invention of the artificial muscle, the motor; and the combination of the motor and tools to machines. And modernism means the transition from belief to certainty, provided by the natural sciences, producing real

world knowledge by inducing categories from observations; and validating theories, by trying to falsify deduced predictions.

Bo: The third question is: What is the root of postmodern thinking?

Paul: As I see it, Lyotard, is one of the first to use of the term 'postmodernism' with reference to philosophical discourse in his book The Postmodern Condition. Lyotard considers all of human knowledge to consist of narratives, whether it is in the traditional narrative forms, such as literature, or in the scientific disciplines. Each disciplined narrative has its own legitimation criteria, which are internal, and which develop to overcome or engulf contradictions. However the roots of postmodernism can be traced to Nietzsche too. When he said 'God is dead' he meant that the days of all absolutes were over – and absolutes as whet underpins modernism.

Allan: To me, postmodernism means Skepticism toward hidden patronization according to Lyotard's statement: 'Simplifying to the extreme, I define postmodern as, incredulity, toward metanarratives'. Skepticism is as old as the republic, beginning in the ancient Greek republic with the sophists, and continuing in the Enlightenment Century. Today skepticism is expressed in the two Enlightenment republics, in the American with pragmatism and grounded theory; and in the French with the post-structural thinking of Derrida, and Lyotard, and Foucault, and Bourdieu.

Bo: The fourth question is: Who is the most important postmodern thinker?

Paul: To me, the most important theorist is Wittgenstein. Wittgenstein made the transition from modern to postmodern thinking in his two books. First in Tractatus Logico-Philosophicus he tries to finish the modern project by showing how the outside world has created our language to represent it. Then in Philosophical Investigations he turns around and shows how in return it is language games that construct the outside world. Thus with his own person and his own work Wittgenstein is the first to realize that the world is not creating language, but created by language. But this language use – in what he terms language games – is based in everyday living in what he terms 'forms of life'.

Allan: To me, the most important theorist comes from the threatened republic, the French. Here I will point to Foucault and his statement when discussing human nature with Chomsky coming from the unthreatened republic, the American. Foucault says: "It seems to me, that the real political task in a society, such as ours, is to criticize the working of institutions, which appear to be both neutral and independent; to criticize them in such a manner, that the political violence, which has always exercised itself obscurely through them, will be unmasked, so that one can fight them. "

Bo: Could you please elaborate?

Paul: Wittgenstein says, mathematical foundations are quite irrelevant to the continued healthy practice of mathematics, both pure and applied. Wittgenstein offers a powerful social vision of mathematics. One of his key contributions is to recognize the social basis of certainty, that following a rule in mathematics or logic does not involve logical compulsion. Instead it is based on the tacit or conscious decision to accept the rules of a 'language game' which are grounded in pre-existing social 'forms of life'. Wittgenstein's importance is to show that that the 'certainty' and 'necessity' of mathematics are the result of social processes of

development, and that all knowledge including that in education presupposes the acquisition of language in meaningful, already existing, social contexts and interactions.

Allan: To me, Foucault shows how human disciplines discipline themselves and their subject. This forces false identities upon humans, who then seek cure at correcting institutions that copy the pastoral power of the Christian church. Furthermore, the institution called education might instead be a place for symbolic violence, that monopolizes society's knowledge capital for a privileged knowledge nobility. Also, institutions are run by people that follow authorized routines, which can create both gas turbines and gas chambers. Following orders might be OK in industry since it is controlled from below by the natural correctness of the market: sell or die. But it may become problematic in institutions that are controlled from above by a political correctness: conform or die. Thus institutionalized patronization might become totalitarian, reintroducing evil actions, rooted not in a devil, but in the sheer banality of just following orders.

Paul: I agree with Allan that Foucault is a very important thinker. His originality lies in his historical analyses of power, institutions and identity, and his rejection of essences underlying everything – from persons and identities, academic subjects and knowledge to names, concepts and ideas. He is usually called a post-structuralist, but post-structuralists share much with postmodernists – most notably the rejection of single monolithic structural theories to explain anything – from society to language.

Bo: The fifth question is: What is mathematics?

Paul: To me, mathematics is what mathematicians do. Mathematics is a language game, or rather a set of language games and forms of life. Mathematics is taught to allow students to take part in some of these language games; because it can be applied in many real world situations and because it has great social and personal power. To get a deeper understanding, again we should listen to Wittgenstein. Mathematics is a multiplicity of practices – Wittgenstein calls it the 'motley of mathematics'. Mathematics provides the entry ticket to many of these practices.

Allan: To me, mathematics is not an action-word, it is a verdict-word that labels or installs what it names. To see, if it labels or installs something, we must ask, which actions are named mathematics? Many, is a natural fact. To deal with Many, we count and add, in short we reckon. Consequently, there is a need for education in reckoning, also called algebra, which in Arabic means to re-unite.

Bo: The sixth question is: What is postmodern mathematics?

Paul: To me, Postmodernism rejects a single authoritative way of seeing mathematics, teachers and learners, for each can be seen and interpreted in multiple ways. Mathematics can be seen as axiomatic and logical leading to indubitable conclusions, but it can also be seen as intuitive and playful, open-ended, with surprises and humor, as evidenced in popular mathematical images and cartoons. Additionally it can be seen in its applications in science, information and communication technologies, everyday life and ethnomathematics. All of these dimensions are part of what makes up mathematics and they all co-exist successfully.

Allan: To me, postmodern mathematics raises the question: Is mathematics education what it says, education in mathematics - Or is it something else, like symbolic violence in meta-matism, which is a mixture of meta-matics, that turns mathematics upside down, by

defining concepts as examples of abstractions, instead of as abstractions from examples; and of mathe-matism that is true in a library, but not in a laboratory: To exemplify, the statement , 2 times 3 is 6, is matte-matics, since 2 threes can be re-counted as 6 ones. Whereas the statement, 2 plus 3 is 5, is mathe-matism, since it has countless counterexamples as e.g. 2 weeks plus 3 days is 17 days.

Bo: The seventh question is: What is postmodern education?

Paul: To me, postmodern education means accepting the diversity in learners' background and interests, in learning material and situations, and in teacher personalities and ethnic background. Allowing playfulness and surprise to enter into education is also necessary. I think mathematics plays a small but significant part in postmodern education. What I would like to see is a truly responsive education system that has a real personal face. Every student should have a relationship with a teacher or other mentor who finds out what the student loves and can achieve real success at. Whether it be some sport, model making, dance, or academic study and creativity, such as in mathematics, it is the responsibility of education to help the student find their own bliss and success. The area that students experience this success in doesn't really matter. Once they have success, enjoyment and self-confidence unleashed by their manifested talent, I believe students can go on to succeed in many other areas of study and life' including mathematics. So the flexibility and truly individualistic element of education is what would make it postmodern, and a great education.

Allan: To me postmodern education means replacing the forced classes and the forced subject combinations of line-organized education aiming at preparing for public offices, with the self-chosen half-year blocks of block-organized education aiming at uncovering and developing the individual talent of the learners.

Bo: The last question is: What is postmodern research?

Paul: To me, all forms of research can fit under the umbrella of postmodernism since it allows for different methods, meanings and interpretations. As it stands research in the interpretative paradigm, is more open to multiple meanings since it accepts that both the world and its description are social constructions in the end. So postmodern research fits less well with the scientific research paradigm with its assumption of one true reality. However, no methods are ruled out by postmodern research, be they qualitative, quantitative or mixed.

Allan: To me, postmodern research was created by the ancient Greek sophists saying: we must enlighten ourselves to tell nature from choice, to avoid being patronized by choices presented as nature. So postmodern research is a search for hidden alternatives to patronizing choices presented as nature.

Bo: Thank you Paul and Allan. It was a nice debate, wasn't it? I learned that you have more in common than differences, although you do differ on some issues.