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FRACTIONS GROUNDED AS DECIMALS, OR 3/5 AS 0.3 5S

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The tradition sees fractions as difficult to teach and learn. Skepticism asks: Are fractions difficult by nature or by choice? Are there hidden ways to understand and teach fractions? Contingency research seaching for hidden alternatives to traditions looks at the roots of factions, bundling the unbundled, described in a natural way by decimals. But why is the unnatural presented as natural?
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THE BACKGROUND

Enlightenment mathematics was as a natural science exploring the natural fact Many (Kline, 1972) by grounding its abstract concepts in examples, and by using the lack of falsifying examples to validate its theory. But after abstracting the set-concept, mathematics was turned upside down to modern mathematics or 'metamatism', a mixture of 'meta-matics' defining its concepts as examples of abstractions, and 'mathema-tism' true in the library, but not in the laboratory, as e.g. 2+3 = 5, which has countless counterexamples: 2m+3cm = 203 cm, 2weeks+3days = 17 days etc. Being self-referring, this modern mathematics did not need an outside world.

However, a self-referring mathematics turned out to be a self-contradiction. With his paradox on the set of sets not belonging to itself, Russell proved that sets implies self-reference and self-contradiction as known from the classical liar-paradox 'this statement is false' being false when true and true when false: If $M = \{A \mid A \notin A\}$, then $M \in M \Leftrightarrow M \notin M$.

Likewise Gödel proved that a well-proven theory is a dream since it will always contain statements that can be neither proved nor disproved.

In spite of being neither well-defined or well-proved, mathematics still teaches metamatism. This creates big problems to mathematics education as shown e.g. by 'the fraction paradox' where the teacher insists that 1/2 + 2/3 IS 7/6 even if the students protest that when counting cokes, 1/2 of 2 bottles and 2/3 of 3 bottles gives 3/5 of 5 as cokes and not 7 cokes of 6 bottles.

Likewise modern metamatism forces natural way to write totals, T = 3.4 tens including the unit-bundle and a decimal point to separates the bundles from the unbundled, to be replaced with plain 34 hiding both the total and the unit, and misplacing the decimal point.

To design an alternative, mathematics can return to its roots, the natural fact Many, guided by contingency research that, inspired by contemporary and ancient skeptical thinking, uncovers hidden patronization by discovering alternatives to choices presented as nature.

CONTINGENCY RESEARCH UNMASKS CHOICES PRESENTED AS NATURE

Ancient Greece saw a controversy on democracy between two different attitudes to knowledge represented by the sophists and the philosophers. The sophists warned that to practice democracy, the people must be enlightened to tell choice from nature in order to prevent hidden patronization by choices presented as nature. To the philosophers, patronization was a natural order since to them all physical is examples of meta-physical forms only visible to the philosophers educated at Plato's academy, who therefore should be given the role as natural patronizing rulers (Russell, 1945).

Later Newton saw that a falling apple obeys, not the unpredictable will of a meta-physical patronizor, but its own predictable physical will. This created the Enlightenment: when an apple obeys its own will, people could do the same and replace patronization with democracy.

Two democracies were installed: one in the US still having its first republic; and one in France, now having its fifth republic. German autocracy tried to stop the French democracy by sending in an army. However, a German mercenary was no match to a French conscript aware of the feudal consequence of defeat. So the French stopped the Germans and later occupied Germany. Unable to use the army, the German autocracy instead used the school to stop enlightenment in spreading from France. As counter-enlightenment, Humboldt used Hegel philosophy to create a patronizing line-organized Bildung school system based upon three principles: To avoid democracy, the people must not be enlightened; instead romanticism should install nationalism so the people sees itself as a 'nation' willing to fight other 'nations', especially the democratic ones; and the population elite should be extracted and receive 'Bildung' to become a knowledge-nobility for a new strong central administration replacing the former blood-nobility unable to stop the French democracy.

As democracies, EU still holds on to line-organized education instead of changing to block-organized education as in the North American republics allowing young students to uncover and develop their personal talent through individually chosen half-year knowledge blocks.

In France, the sophist warning against hidden patronization is kept alive in the post-structural thinking of Derrida, Lyotard, Foucault and Bourdieu. Derrida warns against ungrounded words installing what they label, such word should be 'deconstructed' into labels. Lyotard warns against ungrounded sentences installing political instead of natural correctness. Foucault warns against institutionalized disciplines claiming to express knowledge about humans; instead they install order by disciplining both themselves and their subject. And Bourdieu warns against using education and especially mathematics as symbolic violence to monopolize the knowledge capital for a knowledge-nobility (Tarp, 2004).

Bauman (1989) points out that by following authorized routines modernity can create both gas turbines and gas chambers. Arendt (1998) shows how in highly institutionalised societies patronization might become totalitarian, thus reintroducing evil actions this time rooted not in a devil but in the sheer banality of just following orders.

To prevent patronization, categories is grounded in nature using Grounded Theory (Glaser et al, 1967), the method of natural research developed in the first Enlightenment democracy, the American, and resonating with Piaget's principles of natural learning (Piaget, 1970).

THE CASE OF FRACTIONS

The tradition says that fractions are difficult to teach and learn. Contingency research questions this by asking: Is fraction difficult by nature or by choice? And if so, whose choice? Can hidden alternatives be uncovered? Who has an interest in making fractions difficult?

FRACTIONS IN TEXTBOOKS

The fraction tradition can be observed in textbooks and in books on modern mathematics. Typically, the tradition postpones fractions to after all four basic operations have been introduced. Then unit fractions come in two versions. Geometric fractions are parts of pizzas or chocolate bars. And algebraic fractions are associated with simple division: 1/4 of the 12 apples is 12/4 apples. To find 4/5 of 20 apples, first 1/5 of 20 is found by dividing with 5 and then the result is multiplied by 4.

Then it is time for decimals as tenths, and for percentages as hundredths. Then adding or removing common factors in the numerator and in the denominator introduces the idea of similar fractions. Then, in late primary and early middle school, addition of fractions is introduced, first with like, then with unlike denominators.

Then decomposing a numbers into primes is introduced together with the lowest common multiple and the highest common factor to find the smallest common denominator when adding fractions with unlike denominators.

Then everything is repeated three times. Numerical fractions become algebraic fraction, first using monomials as (4abc)/(6ac) that are already factorized; then using polynomials as (4ab+8bc)/(3ab-6ac) that need to be factorized; and finally fractions enter equations.

FRACTIONS IN MODERN MATHEMATICS

Before modern mathematics, a fraction was a ratio between numbers, not allowing zero in the denominator. Some fractions could be reduced, e.g. 2/4 to 1/2. In modern mathematics they are made the same fraction, or more precisely, different representatives of the same fraction.

Wanting to define everything as examples of sets, modern mathematics defines a fraction as an equivalence set in the set product of ordered pairs of numbers created by an equivalence relation making (a,b) equivalent to (c,d) if cross multiplication holds: a*d = b*c. This definition creates a lot of activities aiming to show how operations defined on equivalence classes are independent upon the actual representatives.

Russell solved his set paradox by introducing type theory allowing only the natural numbers to be numbers, and therefore not accepting a pair of numbers to be a number, less an equivalence set in a set product. To Russell, a fraction is a calculation, not a number. The fractions of modern mathematics therefore violate Russell's set paradox. To make fractions numbers, modern mathematics invented its own set theory, not distinguishing between elements and sets, thus being meaningless by mixing examples and abstractions: you can eat an example of an apple, but not the abstraction 'apple'. Also Peano axioms were invented using a follower principle to prove that 1+1 = 2 is a natural correct statement. However, in a laboratory, 1 week + 1 day = 8 days, and 1 is not well defined since 1 threes = 3 ones, etc.

THE ROOT OF EDUCATION: ADAPTING TO THE OUSIDE WORLD

The prime goal of education is adapting to the outside world by proper actions. A typical action as ‘Peter eats apples’ is described by a three-term sentence with a subject, a verb and an object. Thus mathematics education should be described in this way. The learner is the subject, the object is the natural fact Many, and the verb is what we do to deal with Many, we totalize by counting and adding. Not being a verb, mathematics education should be renamed to ‘totalizing’, ‘counting and adding’ or reckoning. As to fractions, the fundamental question is: what outside world situations root fraction; and which occur in a natural way in primary, middle and high school?

THE ROOT OF MATHEMATICS: COUNTING MANY BY BUNDLING

To deal with Many, we totalize expressing the total as a formula, e.g. T = 345 = 3*10^2 + 4*10 + 5*1 showing that totalizing means counting and adding bundles, and that there are four ways of adding: +, *, ^ and integration. Totalizing can also be called algebra if using the Arabic word for reuniting. It turns out that there are three ways of counting:

1.order counting bundles sticks into icons, so that there are five sticks in the 5-icon etc. 2.order counting bundles the total in icon-bundles, and 3.order counting bundles in tens.

I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9

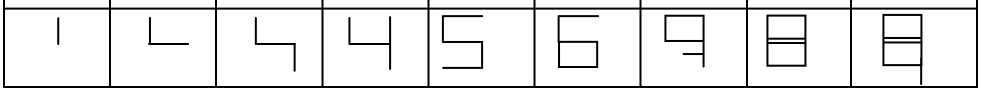


Figure 1. Rearranging sticks into icons transforms 5 1s into 1 5s, etc.

The calculator has no icon for ten since counting in tens, ten becomes 1 bundle, and eleven and twelve, meaning one left and two left in old Norse, becomes 1 bundle 1 and 1 bundle 2, or just 11 and 12, while ten becomes 10 symbolizing 1 bundle and no unbundled. However, ten is not 10 by nature, but by choice of bundle-size. Counting in fives, five becomes 10: 1, 2, 3, 4, 5 = 1 bundle = 10. Thus the icon for the bundle-size is never used.

Operations iconize the bundling and stacking processes. Taking away 4 is iconized as – 4 showing the trace left when dragging away the 4. Taking away 4s is iconized as /4 showing the broom sweeping away the 4s. Building up a stack of 3 4s is iconized as 3x4 showing a 3 times lifting of the 4s. Placing a stack of 2 singles next to a stack of bundles is iconized as + 2 showing the juxtaposition of the two stacks. And bundling bundles is iconized as ^ 2 showing the lifting away of e.g. 3 3-bundles reappearing as 1 3x3-bundle, i.e. as 1 3^2-bundle.

Now numbers and operations can be combined to calculations predicting the counting results. The ‘recount-formula’ T = T/b bs = (T/b)*b tells that from the total T, bs are taking away T/b times. Thus recounting a total T = 3 6s in 7s, the prediction says T = (3*6/7) 7s. A calculator gives the result 2 7s and some leftovers that can be found by the ‘rest-formula’ T = (T–b) + b telling that from the total T, b can be taken away and placed next-to: 3*6 = (3*6–2*7) + 2*7 = 4 + 2*7. The combined prediction, T = 3*6 = 2*7 + 4*1 = 2.4 7s, holds when tested:

IIIIII IIIIII IIIIII -> IIIIIII IIIIIII IIII.

COUNTING THE UNBUNDLED ROOTS COUNT-FRACTIONS

With 2.order icon-counting a total may be bundled in 5s as e.g. T = 2 5s & 3. The sticks are placed in a left bundle-cup and in a right single-cup. In the bundle-cup a bundle is traded, first to a thick stick representing a bundle glued together, then to a normal stick representing the bundle by being placed in the left bundle-cup. Now the cup-contents is described by icons, first using cup-writing 2)1), then using decimal-writing to separate the left bundle-cup from the right single-cup, and including the unit 3s, T = 2.1 3s.

IIIIIIIIIIIII -> IIIII IIIII III -> IIIII IIIII) III) -> ▮ ▮) III) -> II) III) -> 2)3) -> 2.3 5s

Also bundles can be bundled and placed in a new cup to the left. Thus in 6 3s & 2 1s or 6)2) or 6.2 3s, the 6 3-bundles can be rebundled into two 3-bundles of 3-bundles, i.e. as 2))2 or 2)0)2), leading to the decimal number 20.2 3s: III III) II) -> II)) II), or 6)2) = 2)0)2 = 20.2 3s.

Adding an extra cup to the right shows that multiplying or dividing with the bundle-size just moves the decimal point: T = 2.1 3s = 2)1) -> 2)1)) = 2.1 3s * 3 = 21.0 3s and vice versa.

Using squares instead of cups, the 2 5-bundles are stacked and the 3 leftovers may be placed either next to the stack in a stack of 1s, which can be written as 2.3 5s; or on top of the stack of 5-bundles counted as 5s, i.e. as 3 = (3/5)*5 thus giving a total of T = 2 3/5 5s.

Thus 3/5 5s is just another way of writing 0.3 5s. And when bundling in tens, 3/ten tens is the same as 0.3 tens. So for any bundle-number, we can define 'count-fractions' by 3/b = 0.3. And we see that count-fractions are rooted in bundling the leftovers.

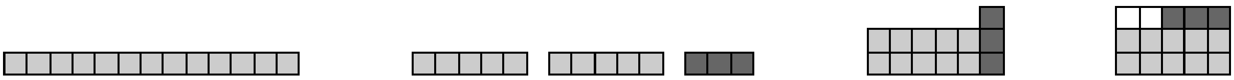


Figure 2. A Total has 3 unbundled added next-to or on-top: T = 2.3 * 5 = 2 3/5 * 5

So count-fractions are a geometrical way to describe the unbundled 1s when counting in bundles. Thus in the case of bundling in 5s or tens, 4 bundles and 3 unbundled can be written as respectively T = 4.3 5s = 4 3/5 5s or T = 4.3 tens = 4 3/10 tens. In this way both 3/5 of 5 and 3/10 of 10 is 3. So to give meaning to 3/5 of 20 we recount 20 in 5s: T = 20 = (20/5)*5 = 4*5 = 5*4 = 5 fours. Thus 3/5 of 20 = 3/5 of 5 fours = 3 4s = 3*4 = 12. Or the short version: 3/5 of 20 = 3 * (20/5).

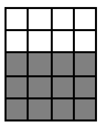


Figure 3. A Total of 20 recounted as 5 4s, thus 3/5 of 20 is 3 4s

Choosing ten as the standard-bundle means that all other bundles must be recounted in tens, which is what the tables does: 4 5s = 4*5 = 20 = 2.0 tens. However, the table can also be reversed recounting tens in e.g. 5s: 20 = (20/5)*5 = 4*5 = 5*4 = 5 fours. Thus taking fractions of numbers allow tables to be practiced both ways. In this way 'fold-numbers' can be folded or factorized partially or fully in non-foldable prime numbers: 12= 3*4= 3*2*2.

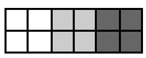


Figure 4. The fold-number 12 folded as 3 4s and as 3*(2 2s). So 12 = 3*2*2.

SHARING A GAIN ROOTS PART-FRACTIONS

Ann and Bo enters a bet with a 5$-stake where Ann pays 3$ and Bo 2$. A gain then should be shared in the same relation 3:2, meaning that the gain should be counted in 5s, each time giving back 3$ to Ann and 2$ to Bo. In this way Ann gets 3/5 of the gain or 3$ per 5$ of the gain. This fraction can be called a part-fraction or a per-number.

ADDING COUNT-FRACTIONS AND PART-FRACTIONS

Once counted, totals can be added. Counting unbundled, count-fractions add the unbundled if the units are the same: T = 1/5 5s + 2/5 5s = 3/5 5s. Different units can be made the same by recounting: T = T1+T2 = 2 1/3 3s +3 1/5 5s = 1 2/5 5s +3 1/5 5s = 4 3/5 5s.

As to part-fractions the totals have to be included when adding. Thus receiving 3/5 of 20$ and 2/3 of 30$ means receiving 3*(20/5)$ + 2*(30/3)$ = 12$ + 20$ = 32$ of 50 $ or 32/50 of 50$.

RECOUNTING IN BUNDLES AND IN TENS

Recounting can unbundle bundles or vice versa. Thus 13.2 5s is the same as 8.2 5s.

T = 13.2 5s = 1)3)2) = 1-1)+5+3)2) = 0)8)2) = 8.2 5s.

When counting in tens 4.5 tens is the same as 0.45 hundreds and 45.0 1s. This can be used when changing units when dealing with amounts, weight and distance:

T = 4.35 $ = 4.35 ten dimes = 43.5 dimes = 43.5 ten cents = 435 cents and vice versa.

T = 4.35 m = 4.35 ten dm = 43.5 dm = 43.5 ten cm = 435 cm and vice versa.

Recounting is useful in manual calculations:

Add	2.3 4s + 3.1 4s = 2)3) + 3)1) = 5)4) = 6)0) = 2)2)0) = 22.0 4s 28 + 45 = 2.8 tens + 4.5 tens = 2)8) + 4)5) = 6)13) = 7)3) = 7.3 tens = 73
Subtract	3.1 4s – 1.2 4s = 3)1) – 1)2) = 2)-1) = 2-1)+4-1) = 1)3) = 1.3 4s 52 – 18 = 5.2 tens – 1.8 tens = 5)2) – 1)8) = 4)-6) = 3)10-6) = 3.4 tens = 34
Multiply	3 * 2.3 4s = 3 * 2)3) = 6)9) = 6+2)-8+9) = 8)1) = 2)-8+8)1) = 2)0)1) = 20.1 4s 3* 58 = 3* 5.8tens = 3* 5)8) = 15)24) = 17)4) = 1)7)4) = 17.4 tens = 174
Divide	2.3 4s / 3 = 2)3) /3 = 2*4+3) /3 = 11) /3 = 3 rest 2, so 2.3 4s = 3* 3 + 2*1 48 /5 = 4.8 tens / 5 = 4)8) / 5 = 4*10+8) / 5 = 9 rest 3, so 48 = 9*5 + 3*1

MIDDLE SCHOOL

Middle school sees the introduction of physical quantities and units. Thus to find 3/5, a length or a weight first must be measured to produce a number. Again taking 3/5 of 46.8 cm means recounting in 5 cm, so T = 46.8 cm = (46.8/5)*5cm = 9.36 * 5cm = 5 (9.36 cm), and 3/5 of this is 3 (9.36 cm) = 28.08 cm. Again we can use the shortcut: 3/5 of 46.8 = 3*46.8/5 = 28.08.

Recounting physical quantities may give per-numbers double units. Thus 3kg sugar might give 5$ when recounted in dollars. So if 3kg cost 5 $, the unit price is 5$/3kg or 5/3 $/kg.

This per-number allows shifting units by recounting. Thus a weight of 12 kg can be recounted in 3s, and a price of 40$ can be recounted in 5s:

T = 12 kg = (12/3)*3kg = (12/3)*5$ = 20$, and

T = 40$ = (40/5)*5$ = (40/5)*3kg = 24 kg

PERCENT

The most frequently used per-number is percent. To transform a given per-number to percent, again we use recounting. To transform 3/5 to percent we recount 100 in 5s:

T = 100 = (100/5)*5 = 20 5s = 5 20s, so 3/5 is 3 20s = 60 meaning 3/5 = 60/100 = 60%.

Again we can use the shortcut: 3/5 of 100 = 3*100/5 = 60.

And to find 5% of a number means finding 5/100 of that number. Let us find 5% of 480:

T = 480 = (480/100)*100 = 4.8 hundreds, so 5% of 480 is 5*4.8 = 24

SIMPLIFYING FRACTIONS

Recounting may be used to simplify fractions: In the fraction 4/6 both 4 and 6 can be recounted in 2s: 4 = (4/2)*2 = 2 twos; and 6 = 3 twos. So instead of writing 4/6 = (4 ones)/ (6 ones) we can write (2 twos)/(3 twos) = 2/3. So both above and below the fraction line common factors can be taken out as units and then cancelled out; or vice versa added as units.

ADDING PER-NUMBER FRACTIONS

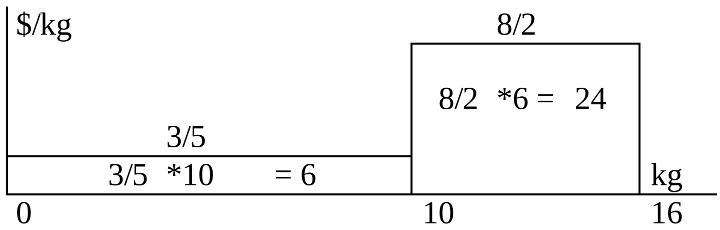
Adding per-numbers occurs when blending tea: If 10 kg at 3$/5kg is blended with 6 kg at 8$/2kg the result is 16 kg at a price that is found by calculating the weighted average:

10 kg at 3/5 $/kg gives 6 $

<u>6 kg at 8/2 $/kg gives 24 $</u>

16 kg at u $/kg gives 30 $

So u = 30$/16kg = 1.88 $/kg



The problem is that where unit-numbers can be added directly, per-numbers are added as areas under the per-number graph, i.e. as Σ p*Δx. This is the case no matter if the per-number is a number or a fraction. Adding per-numbers by their area thus roots addition by integration.

STATISTICS ROOTS FRACTIONS

Reporting numbers from questionnaires includes fractions and percentages: Counting boys and girls in a group of two classes P and Q may result in the following 2x2 cross table:

	P	Q	Total
B (boys)	10	10	20
G (girls)	10	20	30
Total	20	30	50

The fraction describing the boys in class P depends of which total is used. Thus the P-boys make 10/50 or 20% of the group, and 10/20 or 50% of the class. Likewise, the Q-girls make 20/50 or 40% of the group, and 20/30 or 67% of the girls.

	P	Q	Total
B	10/50	10/50	20/50
G	10/50	20/50	30/50
Total	20/50	30/50	50/50

	P	Q	Total
B	50%	50%	100%
G	33%	67%	100%

	P	Q
B	50%	33%
G	50%	67%
Total	100%	100%

HIGH SCHOOL

In high school substituting numbers by letters allows letter fractions or algebraic fractions to occur. Again factoring and canceling out common units can simplify such fractions:

$$\frac{a*b*c}{c*d} = \frac{(a*b)\ cs}{d\ cs} = \frac{a*b}{d} \quad \text{and} \quad \frac{a*c + b*c}{c*d} = \frac{(a+b)*c}{c*d} = \frac{(a+b)\ cs}{d\ cs} = \frac{a+b}{d}$$

Middle school per-numbers are piecewise constant. In high school per-numbers are locally constant, continuous. Still, the area under its graph adds variable per-numbers. This area can be found by a graphical calculator, or by using integration, known from horizontal adding stacks next-to in primary school and from calculating weighted averages in middle school.

Middle school cross tables has as the task to go from data to fractions. In high school the task is to translate vertical fractions to horizontal or vice versa. Bayes' formula is introduced, but a more natural approach is to recreate the original data from which any fractions can be found.

In probability fractions describe probabilities leading to the binomial distribution.

In finance adding constant interest percentages r leads to exponential growth y = yo*(1+r)^n.

2/3 3S BECOMING 2/3

In the laboratory both 2/3 3s and 0.2 3s exist as 2 leftovers when counting in 3s. Counting in tens, 2 leftovers become 2/10 10s or 0.2 tens. However, the tradition insists that when counting in tens, the unit is neglected and the decimal point is moved one place to the right, which transforms 2.3 tens to plain 23, and 0.2 tens to plain 2, i.e. 2 1s. In this way counting in tens is mixed up with counting in 1s, which has no meaning since 1 = 1^1 = 1^2 while 1 $\neq$ 10^1 $\neq$ 10^2. Thus counting in 1s makes it impossible to distinguish between 1s, bundles and bundles of bundles. The Romans never realized that 3 1s, III, could be transformed to 1 3s.

The tradition also insists that fractions as 2/3 3s should drop its unit to become 2/3 of 1, i.e. a calculation producing 0.0667 tens or 0.667. Also the tradition insists that 2/3 is a number instead of a calculation. Furthermore the tradition insists that fractions can be added without units as illustrated in the fraction paradox above.

So while 2/3 3s is labeling a physical fact, 2 leftovers when counting in 3s, 2/3 is not labeling, but installing what it mentions, thus installing hidden patronization that must be deconstructed if education shall serve as enlightenment. And deconstruction might lead to the discovery of 1digit mathematics (Zybartas et al, 2005).

THE TWO GOALS OF MATHEMATICS EDUCATION

In mathematics education, fractions can be used for two different purposes.

Mathematics may be seen as a natural science rooted in the natural fact Many. In this case fractions are introduced in grade one parallel with decimals to account for the unbundled when performing 2.order icon-counting. The core activity of icon-counting is recounting using the recount formula T = (T/b)*b showing directly why fractions occur as division. As a counting fraction, 3/5 only has meaning as 3/5 5s or 3/5 of 5. Simple sharing problems motivate seeing 3/5 as a per-number that can be taken of any number, and the ability to recount any number in 5s allows per-numbers to be introduced in primary school. Thus the core part of fractions is learned before entering middle and high school. Recounting also allows tables to be practiced both ways, both as multiplication tables and as factoring tables. Seeing mathematics as Manyology, a natural science about Many, makes it accessible to all.

However, mathematics may also be seen as metamatism, i.e. a self-referring body of knowledge that is meditated by what is called mathematics education. Metamatism allows only 3.order counting in ten-bundles to take place, thus banning both 1.order counting allowing the learners to build the digits themselves with sticks, and second order counting allowing recounting to introduce both fractions and decimals as well as changing units (proportionality) and horizontal addition next to (integration) in early primary school. Instead metamatism says that 2+3 IS 5 and that 1/2+2/3 IS 7/6, in spite of countless counterexamples in the laboratory. Changing mathematics to metamatism makes it accessible only to an elite.

WHY PRESENTING THE UNNATURAL AS NATURAL

In a natural approach, fractions and decimals occur together when performing recounting, the most powerful operation in mathematics, producing natural numbers as decimals with units.

However, the tradition forces Many to be counted only in ten-bundles, and forces numbers to be presented without units and with displaced decimal points. This forces fractions to be postponed to after the introduction of all four operations when, all of a sudden, it is allowed to count in other units than ten, e.g. 7s; and forces decimals to be presented as examples of fractions. Finally, forcing fractions to be added without units creates mathematism.

Why all this force? One answer comes from Bourdieu and his modern version of the ancient Greek question: Should knowledge enlighten people to practice democracy; or should knowledge persuade people to accept patronization by the better knowing philo-sophers?

Bourdieu sees the social world divided into fields where people fight for the capital of the field. With the transition from industrial to information society, both economical and knowledge capital become important. Socialist parties have decentralized economical capital, but the knowledge capital is still centralized enabling also political power to be centralized to a knowledge-nobility whose offspring develops the relevant habitus to be successful in the knowledge field, much like the mandarin system of ancient China. And Bourdieu sees mathematics as specially suited to perform the symbolic violence that monopolizes knowledge to the nobility. Organizing knowledge in forced lines instead of in self-chosen blocks is another effective technique to guard the knowledge privilege (Bourdieu 1977).

To decentralize knowledge, the symbolic violence must be taken out of education by replacing line-organization with block-organization; and taken out of mathematics by replacing self-referring metamatism with mathematics as a natural science that explores the natural fact Many, and that by making 3.order ten-counting second to 2.order icon-counting also makes fractions second to decimals. With its ability to uncover hidden patronization presenting choice as nature, contingency research is an effective means to replace autocracy with democracy in the knowledge field.

CONCLUSION

This paper asked ‘Are fractions difficult be nature or by choice?’ Contingency research showed that fractions are difficult by choice. The nature of fractions is to account for the leftovers when counting in bundles. Introducing 2.order icon-counting before 3.order ten-counting shows that natural numbers always include units and a decimal point to separate the bundles form the unbundled. So fractions are natural parts of most counting or recounting results. However the tradition allows only 3.order ten-counting to be practiced and forces natural numbers to take on a unnatural identity as multi-digit numbers leaving out the unit and misplacing the decimal point one place to the right so that fractions are no longer needed to account for the unbundled. Instead 2.order counting is postponed to the end of primary school where all of a sudden is it allowed to count in icons to motivate the introduction of fractions. However, forcing fractions to be added without units transforms meaningful mathematics into meaningless metamatism repelling most learners. Except for the children of the knowledge nobility using mathematics and fractions as means to guard its knowledge-monopoly. To decentralize the knowledge capital, hidden patronization should be unmasked by research uncovering hidden contingency to choices presented as nature.

References

Arendt, H. (1998). *The Human Condition*. Chicago: University of Chicago Press.

Bauman, Z. (1989). *Modernity and the Holocaust.* Cambridge UK: Polity Press.

Bourdieu, P. (1977). *Reproduction in Education, Society and Culture.* London: Sage.

Glaser, B. G. & Strauss, A. L. (1967). *The Discovery of Grounded Theory*. NY: Aldine de Gruyter.

Kline, M. (1972). *Mathematical Thoughts from Ancient to Modern Times.* NY: Oxford University Press.

Piaget, J. (1970). *Science of Education of the Psychology of the Child*. NY: Viking Compass.

Russell, B. (1945). *A History of Western Philosophy*. NY: A Touchstone Book.

Tarp, A. (2004). *Pastoral Power in Mathematics Education.* Paper accepted for presentation at the Topic Study Group 25, ICME, 2004. http://mathecademy.net/Papers.htm.

Zybartas, S. & Tarp, A. (2005). One Digit Mathematics. *Pedagogika* (78/2005). Vilnius, Lithuania.