Deconstructing PreSchool Mathematics

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This video on early childhood mathematics shows that children can learn linearity, integration and solving equations when allowed to count in icons less than ten, e.g. in 3s or 5s. To be added on-top, a unit must be changed, later called proportionality or linearity. Adding next-to as 8s is later called integration. And reversed addition is solving equations. Script and screens can be found on MATHeCADEMY.net/videos.

YouTube: http://youtu.be/qgCwVZnALXA

Screen 1

Bo: Welcome to the MATHeCADEMY.net Channel. My name is Bo. Today we address the question: Can Preschool children learn mathematics? And welcome to our guest, Allan, who uses contingency research in his work.

Allan: Thank you, Bo.

Bo: Allan, what is contingency research?

Allan: Contingency research looks for alternatives to choices, that are presented as nature, in order to prevent, hidden patronization.

Bo: Thank you, Allan. Do you have a short answer to today's question?

Screen 2, 3

Allan: Well, Bo. Mathematics was created as a natural science, about Many. To deal with Many, first we count Many, then we add Many. In school we only count in tens, and add on-top. In preschool, learning and playing go together. So here we can also count in icons, i.e., in numbers less than ten. And here we can add both on-top, and next-to.

Bo: But why count in other numbers than ten? And why add next-to?

Allan: Because counting in different numbers means shifting units, which is another word for proportionality or linearity. And adding next-to is another word for integration. Furthermore, reversing addition is just another word for solving equations. And proportionality, integration, and solving equations, are core subjects in mathematics.

Bo: Can you please give some details, Allan?

Screen 4

Allan: Certainly, Bo. First we rearrange sticks to form icons. Thus there are five sticks in the five-icon, if written in a less sloppy way. With this 'first order counting´ each number gets its own icon, until ten. Changing five sticks into a five-icon transforms five ones into one fives, that can be used when counting in fives, by bundling and stacking a total in bundles of fives. So what we can call second order icon-counting counts in icon-bundles. Whereas third order counting counts in tens, as does the school.

Bo: But why does the number ten not have an icon?

Screen 5

Allan: Because we don’t use the bundle-number, when counting. Counting in fives we say: one, two, three, four, bundle; one bundle and one, one bundle and two, etc.. So we never use the word five or the icon five when we count in fives. Likewise when we count in tens.

Bo: Can you give an example of icon-counting?

Screen 6

Allan: Let us take a total of seven sticks and count them in fives. We count by taking away five-bundles, which leaves two sticks unbundled. The bundle we represent with a stick in a left bundle-cup. And the two unbundled we place in a right single-cup. Instead of two cups we can use two brackets, one for the bundles and one for the unbundled. Finally we can write down the counting result using cup-writing as T is one bracket two bracket fives; or using decimal numbers as T is one dot two fives, where the decimal point separates the bundles from the unbundled.

Bo: But is it wise to use decimal numbers instead of natural numbers in preschool?

Screen 7

Allan: It is no problem. As we have just seen, the natural way to write a total counted in bundles is as a decimal number, with a unit. So if we recount a total of four eights in tens, we should write that T is three dot two tens. However, the school writes it as three two, thus leaving out the unit, and misplacing the decimal point. How natural is that?

Bo: I guess we can also count in other icons than five?

Screen 8

Allan: Yes, we can count the seven ones in threes, as two bundles and one unbundled, writing that T is two dot one threes. Or in two-bundles as T is three dot one twos. Strictly speaking, we should write that T is one one dot one twos, since here two two-bundles is the same as a bundle of two-bundles.

Bo: But bundles of bundles, isn’t that too abstract for preschool?

Screen 9

Allan: It probably is. And there is no harm in overloads that leave some bundles unbundled. Thus we can leave a total of four dot one threes as it is, or we can recount it as three dot four threes or as two dot seven threes, since one threes can always be changed to three ones, and opposite.

Bo: So recounting totals in different units is a typical task in preschool mathematics?

Screen 10

Allan: Indeed it is. As an example we can ask the question: How many sevens are five threes? Of course, we can lay out three sticks five times and then re-bundle them in sevens. Then we get the answer, that five threes is the same as two dot one sevens. However, we could also use a calculator to predict the result.

Bo: A calculator in preschool??

Allan: Yes. We count in sevens by taking away seven many times, and the calculator has a horizontal stroke to symbolize taking away one sevens; and an uphill stroke to symbolize wiping away many sevens. The question, we ask the calculator, is, from five threes, how many times can we wipe away sevens? So we enter five times three divided by seven, and the answer is two dot something. To find the leftovers, we ask the next question to the calculator, which is, from five threes, we drag away two sevens, and is left with what? So we enter five times three minus two times seven and get the result one. Thus the calculator predicts that five threes can be recounted as two dot one, sevens.

Bo: Normally calculators and formulas go together. But formulas, isn't that too early for preschool?

Screen 11

Allan: Not necessarily so. A total of 8 is counted in 2-bundles by the action: from the total 8 take away twos 8 over 2 times. This action can be abbreviated to a rebundle- or recount-formula, 8 is 8 over 2 times 2. Or with letters: T is T over b times b. And from a stack of 8 we can remove a bundle of 2 from the top, and place it next-to the remaining stack. This action can be abbreviated to a restack-formula 8 is 8 minus 2 plus 2. Or with letters: T is T minus b plus b. And as we have just seen, recounting a total in bundles means acting these two formulas: First we bundle the total in bees, then we take these bundles away, to get the rest. And, if we replace the two actions with the two formulas, the calculator gives the same answer.

Bo: So once counted, totals can be added?

Screen 12

Allan: Yes. Counted totals can be added both on-top and next-to. With on-top addition, the units must be the same. So if we want to add the two totals two threes and three fours, we recount the two threes in fours as one dot two fours, giving a grand total of four dot two fours. And shifting units is just another word for proportionality or linearity.

Bo: What do you mean with adding next-to?

Screen 13

Allan: If we place threes and fours next-to each other, they are added as sevens. To find the total, first we add the two areas, two times three, and three times four, then we count the total area in sevens. And adding areas by combining multiplication and addition is what integration is about.

Bo: Allan, you also talk about solving equations?

Screen 14

Allan: Yes Bo. Solving equation is just another word for reversed addition. I will give you three examples. First let us look at the question: What is it that together with 3 gives nine? If instead of the question mark we use an x, then we get an equation. Using the restack-formula #on 9 we restack nine as nine minus three plus three. Thus x is nine minus three. So by removing what was added, we see that the solution to the equation x plus three gives nine is the number nine minus three.

Bo: And the second equation?

Allan: The second equation comes from asking e.g. how many twos give a total of eight? Using the rebundle-formula we rebundle eight in twos as eight over two times two. Thus x is eight over two. In both cases we see that moving a number to opposite side with opposite sign solves an equation.

Bo: And the third equation?

Screen 15

Allan: The third equation comes from reversing integration by asking e.g. 1 4s and how many 2s give a total of 2 6s? To find the answer, first we take away the one fours from the two sixes, and then we recount the remaining in twos. So a calculator can predict reversed integration, by reversing multiplication and addition to subtraction and division, which later is called differentiation.

Bo: Allan, it seems as if the order of operations is different in preschool and in school?

Screen 17

Allan: Indeed it is, Bo. The traditional order is addition, subtraction, multiplication and then division, which normally is considered the most difficult. However, totals must be counted, before they can be added. So in the natural order, division and multiplication come first, describing the taking away and stacking of bundles. Subtraction occurs if changing one stick in the bundle-cup to many sticks in the single-cup. With two different kinds, addition is the difficult operation. And both on-top and next-to addition requires recounting, to make the units the same or to find the total area.

Bo: Allan, you have not mentioned fractions?

Screen 16

Allan: Well Bo. Using decimal fractions or ordinary fractions only depends on, what you do with the unbundled. Recounting two threes in fours gives one bundle and two unbundled that added next-to gives the decimal fraction one dot two fours; and that added on-top counted in fours as 2 over 4 fours give a total of one and two over four fours, here describing the unbundled with an ordinary fraction. So both decimal fractions and ordinary fractions occur in a natural way, when recounting totals in icons.

Bo: What happens when these preschool children begin ordinary school?

Screen 3, 19

Allan: Ordinary school mathematics only allows third order counting in tens. If all numbers have the same unit, we never change unit, and we never add next-to. This prevents children from learning about proportionality, integration and solving equations in primary school. To realize these golden learning opportunities, progressive schools will begin primary school with icon-counting and next-to addition. And use dices or dominos to practice recounting predicted by a calculator, as e.g. four threes gives how many fives? And also use colors or plastic letters as coded numbers raising questions as e.g. If c is 3 and s is 5, then a total of 4 C's and 5 S's gives how many C's? Other proposals can be found in my book 'Mathematics as a Natural Science about Many' showing how primary and secondary mathematics become easy when grounded in its roots, the natural fact Many.

Bo: Allan, you have talked very little about two and three digit numbers?

Allan: Well, Bo. Let us write a three digit number as four hundred fifty six as we say it, i.e., as four bundles of bundles plus five bundles plus six unbundled ones. Then we see that any number consists of a combination of one digit numbers and bundles. Multi digit numbers are abbreviations used after school, but why should they be forced upon childeren from their first school year?

Bo: Thank you, Allan, for sharing with us your view on preschool mathematics. Next time on the MATHeCADEMY.net channel we look at year-one and year two mathematics.