Deconstructing PreCalculus

Allan Tarp, the MATHeCADEMY.net, June 2012

This YouTube video on deconstruction in mathematics education connects preCalculus to its root, the natural fact Many. To deal with Many, we total. Some totals are constant, some change in space or time. The change might be predictable, or not. Pre-calculus describes predictable constant change. Calculus describes predictable changing change.

Keywords: postmodern, deconstruction, mathematics, calculus, change, equation, per-number

YouTube link: http://youtu.be/3C39Pzos9DQ

Bo: Welcome to the MATHeCADEMY.net Channel. And welcome to our series on postmodern deconstruction in mathematics education. My name is Bo. Today we address the question "PreCalculus IS hard - or is it?" And welcome to our guest, Allan, who uses postmodern deconstruction in his work.

Allan: Thank you Bo.

Bo: Allan, what does deconstruction mean to you?

Allan: To me, deconstruction means what it says, destruction and reconstruction. It is the method that is used by postmodern skeptical thinking that dates back to the ancient Greek sophists. The sophists warned against patronization that is hidden in choices that are presented as nature. So to avoid hidden patronization, false nature must be unmasked as choice. And to deconstruct then means to discover or to invent alternatives to choices that are presented as nature.

Bo: And what does postmodern mean to you?

Allan: It seems to me that we must distinguish between post-modernism and post-modernity. Post-modernism is what we do with our head, i.e. how we think about the world. And post-modernity is what we do with our hands, i.e. how we act in the world. To simplify, postmodernism is skepticism toward hidden patronization. And post-modernity is the social condition that was created by IT, information technology.

Bo: Thank you, Allan. Do you have a short answer to today's question?

Allan: To me, the short answer to the question "PreCalculus IS hard - or is it" is that preCalculus is not hard by nature, but by choice. PreCalculus is made difficult by its missing link to the root of mathematics, the natural fact Many.

Bo: So, Allan, what is the root of preCalculus?

Screen: Many -> Total, constant, changing -> Change, predictable or not -> Change, constant or changing.

Allan: Well Bo, to deal with Many, we total. Some totals are constant, some change. Some changes are constant, some change. Pre-calculus is about constant change that is predictable. Calculus is about changing change that is predictable. And statistics is about unpredictable change.

Bo: Allan, do you have a short answer to how to make pre-calculus easy?

Allan: Yes Bo, I have a three-step answer. First we look at the two natural examples of constant change. When saving at home the change-number is constant, and adding a number a to b x times gives the number y = b+a*x. When saving in a bank the change-percent is constant, and so is the change-multiplier since adding 5% to b means multiplying b with 105%. And multiplying b with a x times produces the number y = b*a^x. This gives the two basic formulas for constant change.

Bo: And what will be your second step?

Allan: Now we look at formulas with 1 or 2 unknown number, also called equations and functions. Equations we solve by reversed calculation, i.e., we isolate the unknown number by moving the known numbers to the opposite side with opposite sign. Functions we graph by setting up tables for related values of the two unknowns, x and y. Or we use a cheap graphical calculator as TI-82. Here we simply enter left hand side as Y1 and the right hand side as Y2 and find the unknown by pressing solve Y1-Y2 = 0; or by finding the intersection between the two graphs, which is where the left and right hand side numbers are equal.

Bo: And what will your third step then be?

Allan: Finally, to connect mathematics to its root, the natural fact Many, we model real world tables with formulas by using the calculator's regression facility. In this way real world problems are solved by cooperation between humans and technology: Humans ask the question and evaluate the answer giving by technology.

Bo: OK, Allan. Now let us take the details. With many dropouts, what will you do in your first class?

Screen: 456 as we say it, i.e. as T = 4 hundreds + 5 tens + 6 ones = 4*10^2 + 5*10 + 6*1

Allan: I will ask the class to write out a total of 456 as we say it, i.e. as T = 4 hundreds and 5 tens and 6, which is the same as T = 4 ten-tens and 5 tens and 6 ones. Now we see, that we count by bundling in tens, ten-tens etc. And we see that all numbers carry units: hundreds, tens, ones. Also we see that a number is really a formula containing four different ways to unite numbers: By multiplication as 5*10, by power as 10^2, and by addition, which normally is written as 4+5. But here the units are different, transforming the hundreds and the tens into rectangular blocks that cannot be added on-top, but must be added next-to each other. And adding next-to is called integration.

Bo: So there are only four ways to unite numbers?

Allan: Precisely! There are four ways to unite numbers since there are four kinds of numbers in the world: Constant and changing unit- and per-numbers. Unit-numbers carry single-units as meters or seconds, and per-numbers carry double-units as meter/second or meter/100meter = %. So pre-calculus deals with the first three ways to unite numbers: addition, multiplication and power. And calculus deals with the fourth way to unite numbers, which is integration.

Screen:

Bo: Are you going to introduce the traditional names, algebra and geometry?

Allan: Yes I am, since algebra is the Arabic word for reuniting, i.e. uniting numbers into totals and splitting totals into single numbers; and since geometry means earth measuring in Greek, which is done by splitting it up into triangles. And any triangle can be split into right-angled triangles, which can be seen as a rectangle halved by its diagonal. So a right-angled triangle has three angles and the three sides: the base, the height and the diagonal. And the goal of trigonometry is to relate angles and sides with formulas as sine, cosine and tangent expressing one side in percentage of another, thus sinus to angle A is the height in percent of the diagonal.

Bo: And what do you say when the class asks: Why learn mathematics?

Allan: I will point out that we have two languages to describe the world, a word-language and a number-language, and mathematics is really just another word for our number-language. The difference is that the word-language expresses opinions, whereas the number-language predicts.

Bo: Allan, can you please specify?

Allan: Certainly, Bo. Operations are invented to predict numbers. Thus 3 plus 5 predicts the result of adding 1 to 3 5 times. And 3 times 5 predicts the result of adding with 3 5 times. And 3 to the power of 5 predicts the result of multiplying with 3 5 times. And adding 2 3s and 4 5s as areas predicts the number of 8s. So calculations predict the total resulting from a uniting process. A formula combines operations. And a graphical calculator is a typewriter for the number-language used to set up tables for formulas, and to set up formulas from tables. So mathematics is useful because formulas predict.

Bo: But isn't function the basic concept of mathematics?

Allan: The two basic concepts are equations and functions. An equation is a formula with one unknown number. And a function is a formula with two unknown numbers. An equation is solved and a function is graphed.

Bo: But solving equations is hard, isn't it?

Allan: Not if you use a graphical calculator . First we enter the left-hand side as Y1 and the right hand side as Y2. If we want to find the y-number we just enter the known x-number after the Y-number. And if we want to find the x-number, we just solve the equation Y1-Y2 = 0 that just has to be entered once. Instead of using algebra we can use geometry by locating the intersection point between the two graphs, which is where the two numbers on the left and right hand side are the same.

Bo: So students only learn to solve equations on a calculator?

Allan: No. Since equation is just another word for reversed calculation, it is easy to also use the head.

BO. Can you please specify that?

Screen 31

Allan: In a formula we can go two ways. Going forward we can ask: two plus three gives what? Going backwards we ask: two plus what gives five, which we write as the equation 2 + x gives 5. Instead of trying out alternatives we can predict the result by inventing an inverse operation to addition, called subtraction or minus. This allows the answer to be predicted by the calculation five minus two. So by definition five minus two is the number x that added to two gives five. So, an equation is just another word for reversed calculation. And since the equation 2 + x gives 5 is solved by the number x equal 5 minus 2, we see that solving an equation means isolating the unknown by moving numbers to the opposite side with the opposite sign.

Bo: Does this also apply to the other operations?

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Allan: Indeed it does. Since 6/2 is defined as the number x that multiplied with two gives six, we see that the equation x*2= 6 again is solved by moving the number two to the opposite side with opposite sign, i.e., by x = 6/2. Likewise, the equation x to the power of 3 gives 8 is solved by moving the exponent 3 to the opposite side as the third root; and the equation 2 to which power will give eight is solved by moving the base 2 to the opposite side as the 2-based logarithm. Finally the last of the four basic operations, integration, is moved to the opposite side as its opposite operation, differentiation. Again we see that calculations predict numbers.

Bo: So equations can be solved both manually and by using a calculator?

Allan Yes. And to report the work we can use a formula table, i.e., a 2x2 formula table where the first column is for the numbers with the unknown number above the line and the known numbers below. The second column is for the calculations with the predicting formula above the line being transformed to an equation by inserting the known numbers below the line. Now the equation can be solved using the head to move tye known numbers to the opposite side with opposite sign, or using algebra to solve the equation Y1-Y2 = 0; or by using geometry to identify intersection points. To check if the solution is correct, a test is performed inserting the numbers to see if the left-side number is equal to the right-side number.

Bo: And your third step is modeling?

Allan: Yes, mathematical modeling has four steps: First humans ask questions to a table, then technology transforms the table to a formula, then technology provide the answers to the formula questions; and finally humans translate these answers to the real world to be evaluated. To exemplify: Given that we have 10$ after 4 days and 16$ after 9 days, then we can ask what we have after 12 days, and when we have 30$, assuming tat the change will be constant in number or in percent.

Bo: But how do you get the formula?

Allan: As we have seen, changing with a constant change-number is predicted by the formula y = b + a*x, which also can be called linear change. And changing with a constant change-multiplier is predicted by the formula y = b*(1+r)^x, which also can be called exponential change. And if we use an unspecified number instead of ten in the number-formula, then we get what is called a polynomial with a degree, e.g. degree two: T = a*x^2 + b*x + c*1.

Bo: But how about the specific formula fitting the specific table?

Allan: It is difficult to set up without a TI-82, but regression makes it easy to transform a table to a formula. A two-line table only has one change, so as regression formula you can choose between constant change-number and constant change-percent. In a three-line table there are two different changes. So here we can use a polynomial of degree two that is graphed as a bending line called a parabola, having a turning point that can be found by technology. Likewise, a four-line table can be modeled by a degree three polynomial having a double parabola as graph that might have two turning points.

Bo: Allan, can you give examples?

Allan: Certainly Bo. The project 'Population versus food' looks at the Malthusian warning: If population changes in a linear way, and food changes in an exponential way, hunger will eventually occur. The model assumes that the world population in millions changes from 1590 in 1900 to 5300 in 1990 and that food measured in million daily rations changes from 1800 to 4500 in the same period. From this 2-line table regression can produce two formulas: with x counting years after 1850, the population is modeled by Y1= 815*1.013^x and the food by Y2= 300 + 30x. This model predicts hunger to occur 123 years after 1850, i.e. from 1973.

Bo: I guess all tables are two line tables on this level?

Allan: Not necessarily. The project 'Fundraising' finds the revenue of a fundraising assuming all students will accept a free ticket, that 100 students will buy a 20$ ticket and that no one will buy a 40$ ticket. From this 3-line table the demand is modeled by a degree 2 polynomial Y1 = .375*x^2 – 27.5*x + 500. Thus the revenue formula is the product of the price and the demand, Y2 = x*Y1. Graphical methods shows that the maximum revenue will be 2688 $ at a ticket price of 12$.

Bo: Will you give an introduction to calculus?

Allan: I will, both through an example and in general. In the project 'Driving with Peter' his velocity is measured five times. A model can answer many questions, e.g. when was Peter accelerating? And what distance did Peter travel in a given time interval? From a 5-line table the speed can be modeled by the degree 4 polynomial Y1 = -0.009x^4 + 0.53x^3 − 10.875x^2 + 91.25x − 235. Visually, the triple parabola fits the data points. Graphical methods shows that a minimum speed is attained after 14.2 seconds; and that Peter traveled 115 meters from the 10 th to the 15 th second.

Bo: So calculus is about finding turning points?

Allan: Geometrically yes. Algebraically, calculus is about adding per-numbers. Calculus occurs first time in grade one when adding two totals that are counted, not in tens, but in icons less than ten.

Thus if we want to add one fourth and two threes we can do it in two different ways. We can add them on-top as 'hard algebraic addition'. Then the units must be the same, so we recount the 2 3s to 1.2 4s. Thus the total is 2.2 4s. Or, we can add them next-to each other as 1.3 7s. And adding next-to each other as geometrical stacks or blocks is in fact integration.

Bo: But adding blocks and adding per-numbers, is that the same?

Allan. As a matter of fact it is since per-numbers must be transformed to blocks before adding them next-to each other. Thus to add 2kg at 3$/kg and 4 kg at 5$/kg we can add the unit-numbers to 6 kg. But we cannot directly add the per-numbers. To add the per-numbers, we transform them to unit-numbers by multiplication. This gives the area under the per-number graph. Thus the 6 kg cost 2 times 3$ plus 4 times 5$, which is 26$. So the total per-number is 26$ per 6kg. This shows that per-numbers are added by their areas, i.e. by integration. Here the per-number was piecewise constant jumping from 3 $/kg to 5 $/kg. However, the same procedure applies if the per-number is locally constant, continuously changing form one to another.

Bo: Allan, I see that in 2001 you wrote an article called Qualitative and Quantitative literature has three genres: fact, fiction and fiddle. What do you mean by this?

Allan: Well Bo. Fact models quantify and predict predictable quantities, as e.g. 'What is the area of the walls in this room?' In this case the calculated answer of the model is what is observed. Hence calculated values from a fact models can be trusted. A fact model can also be called a since- then model or a room-model. Most models from basic science and economy are fact models.

Bo: And what is a fiction?

Allan: Fiction models quantify and predict unpredictable quantities, as e.g. 'My debt will soon be paid off at this rate!' Fiction models are based upon assumptions and its solutions should be supplemented with predictions based upon alternative assumptions or scenarios. A fiction model can also be called an if-then model or a rate-model. Models from basic economy assuming variables to be constant or predictable by a linear formula are fiction models.

Bo: And fiddle models, what is that?

Allan: Fiddle models quantify qualities: 'Is the risk of this road high enough to cost a bridge?' Many risk-models are fiddle models. The basic risk model says: Risk = Consequence * probability. Statistics can provide probabilities for casualties, but if casualties are quantified, it is much cheaper to stay in a cemetery than in a hospital, pointing to the solution 'no bridge'. Fiddle models should be rejected asking for a word description instead of a number description.

Bo. How about mathematical proofs, should they be left out entirely?

Allan: Not necessarily. In geometry introducing sine and cosine allows a simple proof of the Pythagorean Theorem. From the angle B we have two projections: BD = a*cosB og BC = a = c*cos B. So Rh = BD * c = (a*cosB)*c = a*(cosB*c) = a*a = a^2. Likewise Rv = b^2. Et viola: c^2 = Rh + Rv = a^2 + b^2.

Bo: And how about proofs in algebra?

Allan: We have already proved that a constant change-number leads to the formula y = b+a*x and that a constant change per-cent gives the formula, y = b *(1+r)^n, which can be written in another way as y = b*(1+R) where R is the total interest. This gives the proof of another formula connecting the single interest with the total, 1+R = (1+r)^n. Thus we can see teat adding 8% 5 times means adding 40% plus additional 6.9% since 108%^5 = 146.9% giving a total of 46.9% compound interest.

Bo: But real world saving means you bring that same amount to a bank each month?

Allan: Yes and this can be called change by combined adding and multiplying. And it is described by a very simple formula. Depositing single amount of a$ n times at a single interest at r% gives a total amount A$ and a total interest at R%, related by the formula A/a = R/r. So if we deposit 200$ per year in 10 years, first we find the total interst form the formula 1+R = 106%^10 = 179.1%. So the total saving is A = 79.1/6*200 = 2637 $, which is 637$ more that you put in yourself. An amount that then is taxed.

Bo: So this is an example of a formula you do not prove.

Allan: It could be since knowing how to handle formulas allow you to work with any formula from technology or economy re elsewhere. But in this case the proof is simple. Depositing n times the interest r*(a/r) = a$ of a/r $ to an account makes this a saving account containing the total interest A = R*(a/r). Consequently A/a = R/r where 1+R = (1+r)^n.

Bo: But, Allan, how does this three-step pre-calculus differ from traditional pre-calculus?

Screen: Sets -> Functions -> Linear functions -> Polynomials

Allan: Well Bo. Typically the tradition builds upon modern mathematics wanting mathematics to contain weel-proven statements about well-defined concepts. To be well-defined, all concepts is defined as examples of the concept set. Thus a function is defined as an example of a set product, where first component identity implies second component identity. From this definition, the modern tradition defines two special functions as two homomorphisms. A linear functions is defined by the property that f(p+q) = f(p) + f(q) giving the formula f(t) = a*t: And an exponential function is defined by the property that f(p+q) = f(p)*f(q) giving the formula f(t) = a^t.

Bo: But isn't quadratics also a part of traditional pre-calculus?

Allan: It is, because the tradition mixes modern and pre-modern mathematics. Thus a linear function is defined as f(t) = b + a*t. And the exponential function is defined as f(t) = b*a^t. None of these are homomorphisms. Furthermore the tradition presents the linear function as an example of a polynomial, allowing it two also introduce quadratics, using factorization to prove the formulas for its zeros and turning points.

Bo: That since numbers are build as polynomials, it seems natural to put emphasis on degree one and two polymails in precalculus?

Allan: It does, but in reality it is a choice that is presented as nature and that therefore needs to be deconstructed to find its hidden natural alternative. To do so we should listen careful to the names geometry and algebra. Geometry means earth measuring in Greek; and algebra means to unite in Arabic. This shows the two roots of mathematics, how to measure the earth and how to split what it produces. In both cases we are confronted with the natural fact Many. Thus historically mathematics is created as a natural science describing the natural fact Many.

Bo: But is not set and Many the same?

Allan: It seems so. However, many is a natural fact that exist in the universe before and after humans. Whereas set is a concept created by humans. It is tempting to base mathematics on the concepts set since it makes mathematics entirely self-referring with no need for an outside world to validate its statements. However, the ancient Greeks showed that self-reference leads to self-contradiction as demonstrated by the liar-paradox, saying: 'This sentence is false'. This sentence is true if it is false, and vice versa. Inspired by this Russell formulated his famous set-paradox about self-referring sets: Looking at the set of sets not belonging to itself, it is a contradiction that a set belongs to this set if and only if it does not. To solve this paradox, Russell introduced his type-theory stating that examples and abstraction must never be mixed up. This is illustrated by the fact that you can eat an example of an apple, but you cannot eat the abstract word apple.

Bo: But didn't mathematics find a new way to define sets?

Allan. It did, but in this new definitions sets and elements are mixed up, meaning that examples and abstractions are mixed up, which violates Russell's type theory.

Bo: But according to this theory a pair of numbers is not a number. And a fraction is defined as a pair of numbers, and also a fraction is a number. So Allan, doesn't fractions falsify Russell's type theory?

Allan: Well Bo. The problem here is that a fraction is not a number, a fraction is an operator that need a number to become a number. In other words, a fraction is always a fraction of something, e.g. 2/3 of 6, which is four, whereas 2/3 of 30 is 20. This shows that 2/3 by itself is not well defined.

Bo: But Allan, doesn't a radtional prepacculus begion with revision hwo to add fractions?

Allan: It does, and that is how dropouts remain dropouts as illustrated by the fraction paradox:

The teacher instist that ½ + 2?3 IS 7/6 even if thes stutdent protast by saying that ½ of tow bottles + 2/3 of 3 botyels gives a total of 3/5 of five bottles, and not 7 of 6 bottles.

Bo: But wehy would teachers teach such nonsense in calss.

Allan: Because to stay in office you must conform to the tradition.

Bo: But clearly the tradtion cannot be contradicted by the realties.

Allan: Not if mathematics was a natural sciece about the natural fact Many. However, the tradition has made mathematics si made into a seklfrdrring ndsicourse thjat sustain social power relations.

Bo: Can you pleas specify?

Allan: The core of postmodern scepical thinling is to prevent hidden patronization by choices presented as nature as pointed out by the ancient greek sophists arufng that in a republice ther should be no hidden patronizatin. Perople should enlighten themselves to tell the difernce bwteen chice and nature to prevent being patronize by chces presented as nature. Later the Enlighten ment republec created two republics, an Marixan and a French. The US stil has their frist republic, but the Fench has their fifth. Hence in France Boudriu wanrns aginst societies with ahigh degree og public instrition needin a social class of mandarins to be run. And thes mandarins are selcted by the educational system, which they want to preserve to secure that their mandarin children inher the madarin jobs. And mathematics is expacially usefull here.

looking at the set of stes that belong to itself, a given set is an element if and only if its not an elemnet.

Bo: This might be so histocally, but today mathematics has found a solid definition of a set?

Allan: The problem with todays definition is that it remove dthe difernce betwten elements and sets, that is the diference between exmples and abstractions. This creates another problem as

Bo: So wat is the main differnce betweene set-mased mathematics and many based mathematics?

Allan. Many-based mathematics defines its concepts as abstractiosn form examples thus working bottom-up or from below., ste based- mathematics defines it concepts as examples from abatctions, thus working from above ot top-down. To distinguish the tw, modern top-down set-based mathematics form above might be called selfreferring meta-matics and griounded or rooted mathematics.

Bo. But does this have an effect in the everyday classroom?

Allan: It has, since replacing top-down metamatics with bottom-up mathematics might make many dropouts drop in again.

Bo: Do you have any evidensc for this hope?

Allan: I can refer to a case-study, which could be called: Saving dropput Ryna woth a TI-82.

Allan: I already have since calculus is the fourth way to unite numbers. Here integration adds per-numbers by their areas. And differentiation is just another word for reversed integration.

Bo: Allan, you also mentioned modeling?

Allan: Yes Bo. As mentioned modeling has four steps. First a real world problem is translated into a math problem that is solved and translated back to a real world solution to be evealuated. Typically, the real world problem is a table containg lines with known x and y and some line wher x or y is unknown. With regression allows the table to be translated into a formula that is graphed. The the solution can be read from the graph or by wsolving equations coming from the formula.

INTRODUCING THE THREE GENRES OF QUANTITATIVE LITTERATURE

and form the grap

Allan: Modeling has three

Modelling with regression

2-line tables: Mailthus

3 line tables: fundraising

4-line tables: Out driving

Bo: Will you also give a short introduction to the difference between calculus and pre-calculus?

Allan: I will. Modeling constant change begins with a two-line table illustrating the change.

Bo: Allan, can you please specify how?

Bo: Can you give details from this case-study?

Allan: I would like to Bo. Before being implemented, the curriculum must be designed by applying what migt be called curriculm arcitacture. The goal of a precallus curriculum is constant changing change. As we know thare are two kinds of schools line-organized schools with forced classes and forced timetables aiming at preparing student for an final exam lading on to a private or publiv office, and block-rganized schools with daily lessons in self-chosen halfyare blocks aiming at uncover and develop the individul talents of the individual student. At line-organized schools many students drop out of mathematics and only take part of the math calss because they have to. Her the challenge is to persused dropouts to drop inagain. Conseqiently mathematics must be rebiul from its root, the natural fact Many.

Bo: But Allan, is ther time to begin from the beginning at the pre-calclus level?

Screen: Many -> Number -> T = 456

Allan: To point is to establish a linmk between pre-callcus and the root of mathematics, the natural fact Many. To dela with Many, we total. Let us write the total 456 as we say it, i.e., as four hundreds, five tens and six ones. We see that the total is not a number but a calculation that adds three stacks: a stack of four ten-bundles of ten-bundles, and a stack of five ten-bundles and a stack of six unbundled ones. However, the three stacks are not added on-top of each other; they are added next-to each other. So in what we can call 'hard algebraic addition' stacks are added on-top of each other, while in what we can call 'soft geometric addition' stacks are added next-to each other. And adding next-to each other as geometrical stacks is in fact integration. Also we see that all numbers carry units: ones, tens, and tens-tens, also called hundreds. And finally we see the four ways we unite numbers: we add, we multiply, we power, and we integrate.

And since uniting is called algebra in Arabic, we can now set up a table showing the algebra project: Since numbers can be constant or changing, and since numbers can be unit-numbers as meter or seconds, or per-numbers as m/s, there are four ways to unite numbers: Addition and multiplication unites changing and constant unit-numbers; and integration and power unites changing and constant per-numbers. This table gives us the four basic formulas, where the first three is used in pårecauculus as shown by the formulas for change with a constant number and a constant percentage.

This again leads to introducing equations as reversed calculation

Bo: But where do you meet constant change in your daily life?

Allan: In economy you meet constant change-number when adding additional items at the same price to your basket. In technology you often add the same rate several times, e.g., 8 meter/second added 7 times. And in economy you meet a constant change-percent when saving in a bank, or when an car decreases in value by the same percent per year. In biology a population grows by adding a certain percent per year: And in nature radioactivity decreases with a constant percent per year. Alternatively, constant change can be described by the time it takes to double or half it.

Allan: The seventh missing link is reversed calculation. Going forward we ask: two plus three gives what? Going backwards we ask: two plus what gives five? Instead of trying out alternatives we can predict the result by inventing an inverse operation to addition, called subtraction or minus. This allows the answer to be predicted by the calculation five minus two. So by definition five minus two is the number x that added to two gives five.

Bo: But isn't 2 + x = 5 an equation?

Screen 31

Allan: Well, an equation is just another word for reversed calculation. And since the equation 2 + x gives 5 is solved by the number x equal 5 minus 2, we see that solving an equation means isolating the unknown by moving numbers to the opposite side with the opposite sign.

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Allan: Indeed it does. Since 6/2 is defined as the number x that multiplied with two gives six, we see that the equation x*2= 6 again is solved by moving the number two to the opposite side with opposite sign, i.e., by x = 6/2. Likewise, the equation x to the power of 3 gives 8 is solved by moving the exponent 3 to the opposite side as the third root. And the equation 2 to which power will give eight is solved by moving the base 2 to the opposite side as the 2-based logarithm. Finally the last of the four basic operations, integration, is moved to the opposite side as its opposite operation, differentiation. Again we see that calculations predict numbers.

Now it is time to introduce a 2x2 formula table with the first column is for the nubers with the unknown number above the line and the known numbers below. and the second coum is for the calculations with the predicting formula above the line, being tranfromed to an equation by inserting the nymbers to be solved below the line. To check if the solution is correct, a test is perfomed with alinserting all numbers to see if the lefthand side number is eqial the right hand side.

BO: This aloows students to solve equations manually. Why introduce technology also. Technology is expensice and migt confuse them

Allan: It is impotant to lean to cooperate with technology it gives additional ways to solve equations. Afirst the equatins is enterede. Any eaution has a left hand side

Screens

Now it is time to use regression to set up formulas

Traditional proofs

Bo: Traditional names

Tha are jyst additional named to something known as a foreign language

Now ti is time for modelling.

How dis the students react.

For the frst time all student passed the exam both the written and the oral.

Many engeneers, som dint want to include calculus since they heard that it was very difficult.

But since many students are dropouts

Bo: Thank you, Allan, for sharing with us your view on the question ‘mathematics IS hard – or is it?” In these two sessions we heard about the eight missing links of mandarin mathematics. Next time on the MATHeCADEMY.net channel we will look at fractions. Again we will ask: ‘Fractions is hard – or is it?’

Bo: OK, Allan. So to put it shortly, what is algebra in this connction?

AllaN: Well Bo, algebra is simply an Arabic word for reuniting, anmd since there are forur kinds of numbare sin the world taher are foru way top unite numbers are the opposite of unitin ti to split thare are also four ways to split numbers. As a mater of fact ther are fives since while the order doent matter in adition and smultipliceteion is does in pwering.

Bo: So now it is time to aplly geometry and algebra, I guwess?

Allan: Yse. As to geometry the typival application is to determine a sidstance by imbedding the distance in a triangle, whjre three things can be mausured as e.g findeing the hight of a falgpool by *masuring tweo direction angles from a known baseline. Also angles in the big triangle doesnot have to ve maarured since thehy can be duoind by small atriabgle ehare we nkon the base and the heigtr. Here the tangent number prediucts the angel size, and it is a nice experience to check to see if the priecidtion hold.*