Deconstructing Fractions

Allan Tarp, the MATHeCADEMY.net, July 2012

This YouTube video on deconstruction in mathematics education connects fractions to its root, the leftovers when performing icon-counting. To deal with Many, we total by bundling in icon-numbers less than ten, or in tens needing no icon as the standard bundle. When bundling in 5s, 3 leftovers becomes 0.3 5s or 3/5 5s, thus leftovers root both decimal fractions and ordinary fractions.

Keywords: postmodern, deconstruction, mathematics, fraction, decimal, per-number

http://youtu.be/PtRuk0EWmaQ

Screen 1 & 2

Bo: Welcome to the MATHeCADEMY.net Channel. And welcome to our series on postmodern deconstruction in mathematics education. My name is Bo. Today we address the question "Fractions IS hard - or is it?" And welcome to our guest, Allan, who uses postmodern deconstruction in his work.

Allan: Thank you Bo.

Bo: Allan, what does deconstruction mean to you?

Allan: To me, deconstruction means what it says, destruction and reconstruction. It is the method that is used by postmodern skeptical thinking that dates back to the ancient Greek sophists. The sophists warned against patronization that is hidden in choices that are presented as nature. So to deconstruct means to unmask false nature by finding hidden alternatives to choices presented as nature.

Bo: And what does postmodern mean to you?

Screen 3

Allan: Post-modernism is what we do with our head, i.e. how we think about the world. And post-modernity is what we do with our hands, i.e. how we act in the world. To simplify, postmodernism is skepticism toward traditions hiding patronization by presenting its choices as nature. And post-modernity is the society that is created by IT, information technology.

Bo: Thank you, Allan. Do you have a short answer to today's question?

Allan: To me, the short answer to the question "Fractions IS hard - or is it" is that fractions is not hard by nature, but by choice. Fraction is made difficult by its missing link to the root of mathematics, the natural fact Many.

Bo: So, Allan, what is the root of fractions?

Screen: 4

Allan: Well Bo. To deal with Many, we count by bundling. We can bundle in icon-numbers, or we can bundle in tens, needing no icon since it has been chosen as the standard bundle-number. When bundling in 5s, 3 leftovers becomes 0.3 5s or 3/5 5s, thus leftovers root both decimal fractions and ordinary fractions.

Bo: Allan, do you have a short answer to how to make fractions easy?

Screen 5 & 6

Allan: Yes, Bo. Simply teach first order icon-creation and second order icon-counting instead of skipping both and go directly to third order ten-counting.

Bo: Allan, can you please specify?

Allan: Certainly, Bo. Mathematics is a natural science about the natural fact Many. To deal with many, we total, i.e. we ask the question “how many?” And the answer is given by the total. That is why we use the word algebra that means to unite in Arabic. Thus all mathematics statements should begin with its subject, the total, and specify what the total is. The first step is to represent the total in three ways, by physical sticks, by graphical strokes, and by spoken words. Thus four things can be represented by four sticks on a table; and by four strokes on a paper; and by the word four. Now we can write that T equals stroke stroke stroke stroke.

Bo: That is what the Romans did, isn’t it?

Allan: Indeed it is. But the Arabs went one step further by using icons. They united the four strokes into one single symbol consisting of four strokes. Thus they transformed four ones into one fours.

Bo: But isn’t four ones and one fours the same?

Screen 6

Allan: Well Bo. Four ones means that you count in ones, so that one is the unit. Whereas one fours means that you count in fours, so that four is the unit. And that you count by bundling and stacking the total in fours. If we write the digits in a less sloppy way, we can see that there are five sticks and strokes in the five-icon, and six strokes in the six-icon, etc. So first order counting bundles sticks into icons, so that there are five sticks in the 5-icon etc. And second order counting bundles the total in icon-bundles, where third order counting bundles in tens.

Screen 7

Bo: But why do the icons stop at ten?

Screen 8

Allan: We count by bundling, but we never use the bundle-icon. If we count or bundle in fives, we count one, two, three, four, bundle, one bundle one, one bundle two etc. Or in a shorter way: 1, 2, 3, 4, 10, 11, 12 etc. Thus we do not use the five-icon. Likewise, ten does not need an icon when counting in tens. In this way ten is the only number with its own name but without its own icon. This makes ten a cognitive bomb since ten is the follower of nine while one zero is the follower of four when counting in fives. So ten is not one zero by nature, but by choice of the bundle-number.

Bo: Allan, what do you mean with leftovers?

Screen 9

Allan: Well, Bo. First we count the sticks or strokes by bundling them in e.g. fours. Then we can place the bundles in a left bundle-cup, and the unbundled in a right single-cup. We don’t have to place the physical bundles. For each bundle we just place a stick in the bundle-cup. Now we can use the icons to write the total using a decimal point to separate the bundles from the unbundled. In this way a total of 6 1s can be counted in 4s as 1 4s and 2 1s, and written as 1.2 4s. So a natural number includes a decimal point to separate the bundles from the unbundled, and a unit. Thus a total of 7 1s can be recounted in 5s as 1.2 5s, or as 1.3 4s, or as 2.1 3s, etc. If counted in tens, 7 1s become 0.7 tens. However, we only write 7 leaving out the unit and misplacing the decimal point one place to the right. This may be OK in business, but it creates learning problems in school.

Bo: So decimal numbers come before ordinary fractions.

Allan. They do if we represent Many by sticks or strokes. But they are created at the same time if we represent Many by blocks. Here the bundles can be placed on-top of each other as a stack. As to the unbundled we must make a choice. We can place them next-to the stack, or we can place them on-top of the stack. If placing the unbundled next-to the stack we might have 2 fives and 3 ones, which can be written with decimals as 2.3 5s. Placing the unbundled on-top of the stack we have to count the 3 unbundled in 5s, and the recount formula then gives the result that 3 = (3/5) 5s =

(3/5)*5. So preferring decimal to fractions is a question of taste. In all case the two forms is united by the fact that 0.3 5s is the same as 3/5 of five.

Bo: What do you mean with the recount-formula?

Allan: We saw that digits are icons that contain the number of strokes they represent. Likewise, operations are also icons that show the counting processes they represent. Taking away 4 is iconized as a horizontal stroke showing the trace left when dragging away the 4. Taking away 4 many times, i.e., taking away 4s, is iconized as an up-hill stroke showing the broom sweeping away the 4s. Placing a stack of 4 singles next-to another stack is iconized as a cross showing the juxtaposition of the two stacks. And building up a stack of 3 4s is iconized as an up-hill cross showing a 3 times lifting of the 4s.

Bo: So 3/4 means 3 counted in 4s?

Allan: Precisely! Counting in fours means to repeat taking away four, i.e. dividing the total by four. So the counting result can be predicted by a recount-formula saying that the total T can be bundled in bs T/b times. So T is (T/b) bs, which is the same as (T/b) times b. Thus the recount-formula predicts that recounting a total of 8 1s in 4s gives 8/4 of the 4s, which is 2 4s.

Bo: So recounting is done by de-bundling and re-bundling?

Allan: That is one option. So to recount 4 5s in 6s manually, first we must count up the 4 5s. Then we must debundle them in 1s. And finally we must rebundle them in 6s. This is a long and tiresome job. However, if we use the recount-formula and a calculator, we can predict the result to be 3 6s and a rest. And the rest is found by removing the 3 6s from the 4 5s. So the result of recounting can be predicted by a calculator using division and subtraction.

Bo: From the recount-formula it seems as if division and multiplication come before addition and subtraction?

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Allan: Indeed they do. After recounting is predicted by a rebundle-formula using division and multiplication, we can predict the unbundled by a restack-formula using subtraction and addition. So the natural order of operations is division, multiplication, subtraction, and in the end addition. This is in contrast to the tradition that reverses the natural order, which creates yet more learning problems.

Bo: Allan, how about adding fractions?

Allan: OK, Bo. Let us begin with adding numbers. What would you say is most correct, saying that 2+3 is five or saying that 2 times 3 is 6?

Bo: To me they are both correct, but if I should choose I would say that 2+3 = 5 is most correct.

Allan: Well, I think we should distinguish between grounded mathe-matics that is rooted rooted in observations and ungrounded 'mathe-matism' that is true in the library but not in the laboratory. Thus '2*3 = 6' is natural correct since it is grounded in the fact that with 3 as a unit, 2 3s can be recounted as 6 1s. In contrast to this saying that '2+3 = 5' may be political correct in a library, but has countless counterexamples in the laboratory: 2 weeks + 3 days = 17 days etc. Likewise, real-life fractions are per-numbers to be multiplied with their totals to becomes unit-numbers before being added: 2 kg at 3/4 $/kg + 5 kg at 6/7 $/kg = 7 kg at (3/4*2+6/7*5)/7 $/kg.

Bo: What do you mean with a per-number?

Allan: A physical quantity can be recounted in different units. Thus 3kg sugar might give 5$ when recounted in dollars. So if 3kg cost 5 $, the unit price is 5$/3kg or 5/3 $/kg. This per-number allows shifting units by recounting. Thus a weight of 12 kg can be recounted in 3s, and a price of 40$ can be recounted in 5s:

T = 12 kg = (12/3)*3kg = (12/3)*5$ = 20$, and

T = 40$ = (40/5)*5$ = (40/5)*3kg = 24 kg

Bo: But to add fractions we must teach how to find the common denominator?

Allan: We must indeed if we want to teach mathematism instead of mathematics. The fraction paradox will illustrate the difference. A teacher asks the class: What is 1/2+2/3? The class answers that 1/2 + 2/3 = (1+2)/(2+3) = 3/5. The teacher then says: No. The correct answer is 1/2 + 2/3 = 3/6 + 4/6 = 7/6. To this the class asks: But 1/2 of 2 cokes + 2/3 of 3 cokes is 3/5 of 5 cokes! How can it be 7 out of 6 cokes? The point is that all numbers have units and you can only add if the units are the same. 2/3 does not exist in itself; it will always be 2/3 of something as demonstrated in the recount formula.

Bo: But the recount formula only shows that 2/3 of 3 is two. How about 2/3 of 15?

Allan: Well, you just recount 15 1s in 3s as (15/3) 3s i.e. as 5 threes. So 2/3 of 15 is the same as 2/3 of 3 5 tinmes , that is ten.

Bo: Allan, cam you briefly sum up your view on fractions.

Allan: I will be glad to do so, Bo. Fractions did not cteate themselves, fractions are rooted in the root of mathematics, the natural fact many. Counting many in bundles, leftovers might be palced nect to described as decimale or ontop of the bundle-stack, described as ordibary frations. Thsu both decinmal fractions and ordinary fractions come in naturally in grade one. However, the tradtion insiste that counting takes place in tens. And instead of using the naurale way to represent numbmegers as e.g 3.2 tens, the tradition insiste that the unit is removed and that the decimal point isw misplaced. Thus definmal farctions are hidden until they are introduced in middle school as special fractions, having as denominator the number ten or powers of ten. Likewise, ordinary fractions are postlponed to middle school and defined as a spoecial case of division.

Bo: Thank you Allan, for sharing with us your view on the question: Fractions is hard – or is it? Next time at the MATHeCADEMY.net channel we will look at deconstruction of pre-calculus. Again we will ask: pre-calculus is hard – or must it?