Deconstructing Calculus

Allan Tarp, the MATH**e**CADEMY.net, May 2013

This YouTube video on deconstruction in mathematics education describes how natural calculus is made difficult by its missing link to its root, the addition of per-numbers. Recreating the missing link will make calculus easy and accessible to all.

The YouTube address is: http://youtu.be/yNrLk2nYfaY

Calculus adds per-numbers by areas. Adding 3 kg at 4 $/kg and 5 kg at 6 $/kg, the unit-numbers 3 and 5 are added as 3+5, and the per-numbers 4 and 6 are added as 3*4+5*6, i.e. as the area under the per-number graph, i.e. as integration combining * and +. As reversed integration, differentiation produces per-numbers by combining – and /.

Screen 1

Bo. Welcome to the MATHeCADEMY.net channel. My name is Bo. And welcome to our series called, Mathematics is hard, or is it? Today we will look at Calculus. And welcome to our guest, Allan, who uses deconstruction in his work.

Allan: Thank you Bo.

Bo: Allan, what does deconstruction mean?

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Allan: Well Bo. Deconstruction means what it says, destruction and reconstruction. Deconstruction is used by postmodern skeptical thinking that dates back to the ancient Greek sophists. The sophists warned against patronization that is hidden in choices that are presented as nature. So to avoid hidden patronization, false nature must be unmasked as choice. And to deconstruct then means to discover alternatives to choices that are presented as nature.

Bo: Thank you Allan, do you have a short answer to today's question?

Allan: The short answer to the question, calculus is hard or is it?, is that calculus is not hard by nature but by choice. Calculus is made difficult by its missing link to its root. Reconnecting calculus to its root will it make easy and accessible to all.

Bo: And what is the root of calculus?

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Allan: The root of calculus is adding per-numbers. If we add 3 kg at 4 $/kg and 5 kg at 6 $/kg we get 8 kg at a certain price. Here we add the unit-numbers 3 and 5 to 8. If also we added the per-numbers 4 and 6 to 10, the total price would be 8 times 10, i.e., 80 $, in contradiction to the factual total price, which is 3 times 4 plus 5 times 6, i.e., 42 $.

Bo. So per-numbers cannot be added?

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Allan: They can, but to do so, they first must be changed to unit-numbers by multiplying with their kg-numbers. And multiplying two numbers gives the area of the rectangle having the two numbers as its sides. Thus the total $-number is found by adding areas, which can also be called integration. So 3 kg at 4 $/kg and 5 kg at 6 $/kg gives 3 plus 5 kg, at 3 times 4 plus 5 times 6, divided by 8, $ per kg. Thus integration combines multiplication and addition.

Bo: But shouldn't calculus begin with differentiation?

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Allan: On the contrary. Differentiation is just another word for reversed integration. And you must be able to integrate before you can reverse it. As an example we can ask: 3 kg at 4 $/kg and 5 kg at how many $/kg gives 8 kg at 7 $/kg? When integrated, the area 3 times 4 and the area 5 times the unknown are added to the total area 8 times 7. So to find the unknown, we must subtract the first area from the total area before dividing by 5 to get the per-number. And where integration combines multiplication and addition, differentiation does the opposite it combines subtraction and division.

Bo: So calculus is just about mixing quantities?

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Allan: well, mixing quantities is the main root of calculus. In mixture problems the different per-numbers are constant, which geometrically gives a piecewise constant graph, having the total as the area between the graph and the horizontal base line, the x-axis. However, with a falling body, its m/s number, i.e. its speed, is not constant, but increasing. But the basic question is the same as e.g.: 3 seconds at 4 m/s increasing to 5 m/s gives at total of how many meters? And again the answer is given by the area under the per-number graph.

Bo: But when changing from 3 to 4 meter per second, then the meter per second number is not constant?

Allan: Correct. The per-number is not constant, nor is it piecewise constant, but it is locally constant, and therefore again the local areas add up to the total area.

Bo: What do you mean by locally constant?

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Allan: Let us give a formal definition of being constant. A number y is globally constant c if, for all positive numbers d, the distance between y and c is less than d. Likewise, a variable number y is piecewise constant c, if an interval-radius e exists so that, for all positive numbers d, the distance between y and c is less than d within this radius. Finally we can interchange e and d and define, that a variable number y is locally constant c if for all positive numbers d, an interval-radius e exists, so that the distance between y and c is less than d within this radius. In the same way we can give a formal definition of linearity by saying that the graph of a formula y = f(x) is linear, globally, piecewise or locally, if its change per-number dy/dx is globally, piecewise or locally constant.

Bo: It sounds a little abstract?

Allan: Formal definitions usually are abstract together with the names they define, such as continuity and differentiability instead of constancy and linearity. But what it means is that for any degree of accuracy d, we can find an interval where the distance from y and c is less that the accuracy, i.e., y and c will seem to be the same in this interval. This makes the y-graph piecewise constant, so we can add up areas.

Bo. But with a high degree of accuracy there will be extremely many areas to add up?

Allan: Indeed there will, unless the areas can be written as changes, for adding up a big number of changes gives one single change, which is the difference between the start number and the end number.

Bo: Can you please explain that?

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Allan: Let us set up a five row table with arbitrary numbers in the first column. In the second column we calculate the single changes and add them in the third column. In the fourth column we calculate the total change from the start number to the actual number. We observe that, since the middle numbers always cancel out, adding any number of single changes gives just one change, the total change, which is the difference between the end number and the start number. We use the old S-type for summing up very small changes, dx.

Bo: But how can we write an area as a change?

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Allan: It is straightforward. As an example let us write the area under the curve h = 2x as a change. In the formula y = x squared, a small change in x, dx, will give a small change in y, dy. We can prove, that dy is 2, times x, times dx. Since a local area under h is, h times dx, i.e., 2 times x, times dx, it can also be written as a change in y, dy. So the total area from x = 1 to x = 5 is found by adding up the local changes to one, single change, that is calculated as the end-number, 5 squared, minus the start number, 1 squared, i.e., as 25, minus 1, which is 24.

Bo: OK, Allan, but how can you prove this change formula?

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Allan: In a rectangle with sides f and g, the area A is f times g. If f and g both depend on a third number x, then a small change in x will give small changes in f and in g, called df and dg. And they will give a small change in the area, dA, consisting of three parts, df times g, f times dg, and df times dg. So df times dg is the difference between the number dA and the number df times g plus f times dg. This difference can be made arbitrarily small, so locally, dA is df times g plus f times dg. Using dashes for the ratio of changes, we see, that, f times g, dashed, gives, f dashed, times g, plus, f times, g dashed.

Bo: OK, but you talked about the formula y = x squared?

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Allan: We know that if y = x, then dy/dx = dx/dx = 1, or x dashed = 1. Since we can write x squared as x times x, we see that x squared dashed is x dashed times x twice. Thus we have now proved that a small change in x squared is 2 times x times dx.

Bo: Then how about the area under, e.g., h = x squared?

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Allan: Since x cubed can be written as x squared times x, we see that x cubed dash is x squared dash times x plus x squared times x dash, i.e., 3 x squared. Thus a local area under x squared can be written as a change in one third x cubed.

Bo: So any formula can be written as a change-formula?

Allan: Well, all formulas have change formulas, but not all can be written as a change formula. If not, we have to add up areas, but with a computer this is no problem.

Bo: If differentiation is reversed integration, then reversed differentiation is integration?

Allan: It is, but often it is also called differential equations. Thus finding the area under x squared form x = 1 to x = 4 can be formulated as a differential equation asking: Given that the local change of the area is x squared times dx, then what is the total change in the area from x = 1 to x = 4? Or in other words: Find A(4) by solving the differential equation A' = x^2, given that A(1) = 0.

Bo: Where is calculus used?

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Allan: Physics and economics both use addition of per-numbers. Physics describes the motion of bodies by their velocity measured as meter per second, and by the acceleration, which is the change in velocity per second. Economics talks about the rate of change of the price of a commodity. And talks about the relative rate of change in capitals, or in production, or in a population. And talks about the elasticity, which is the ratio of the relative changes in y and x when describing the demand and supply on a market.

Bo: How old is calculus

Allan: Calculus was invented by Newton when asked to explain the motion of the moon. In the 17th century world trade was about shipping western silver to India in exchange for pepper and silk. The English robbed Spanish silver ships returning from South America, but to get to India they had to navigate by the moon on open sea to avoid the Portuguese fortification of Africa's coast.

Bo: But everybody can see, that the moon moves among the stars?

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Allan: That was what everybody thought until Newton presented his four times no. No, the moon does not move among the stars, as the apple the moon falls towards the earth, in an eternal orbit having the same curvature as the earth. And No, the moon does not follow the Lord's unpredictable will, its motion can be predicted by a formula for a gravitational force. And No, a force does not give motion, a force gives change in motion. And No, algebra cannot solve change-equations, calculus must be invented to do so.

Bo: Allan, you are often quoted for saying that calculus is the fourth and last way of adding. What do you mean by this?

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Allan: Well Bo, mathematics is a natural science about the natural fact Many. To deal with Many, we count and add. Once counted, Many occurs in four different ways: as constant or variable unit-numbers or per-numbers. So numbers can be added in four different ways. Since 3$ and 5$ add up to 3+5 $, variable unit numbers are added by plus. Since 3 $ 5 times add up to 3*5$, constant unit-numbers are added by times. Since 3% 5 times add up to 103% to the power of 5 minus 100%, constant per-numbers are added by power. And since the total of 3 seconds at 4 m/s increasing to 5 m/s is found by adding areas, variable per-numbers are added by integration.

Bo: Also, you are quoted for saying that calculus should be part of primary school?

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Allan: Yes. If two totals 2 3s and 3 4s are added next-to each other as 7s, then they are added as areas, which is integration. In this case the total is 2.4 7s, where we use a decimal point to separate the bundles from the unbundled singles. And if reversed, to find out how many 2s that added to 1 4s gives 2 6s, we first take away the 1 4s and then count the reaming in 2s, which is differentiation.

Bo: Why is integration not part of primary school?

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Allan: Primary school only allows counting in tens, and not counting in icons less than ten. Likewise, it only allows totals to be added on top, and not next to. Instead preschool should take advantage of golden learning chances presented by icon counting and next-to addition, since they allow the children to learn linearity when shifting units, and integration when adding next to, and equations when reversing addition.

Bo: Allan, do you have a conclusion?

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Allan: Yes, Bo. When calculus is deconstructed, a hidden alternative turns up, addition of per-numbers. High school calculus will change from hard to easy if built upon mixing quantities in middle school and upon adding next-to in primary school. Having once reversed integration by taking apart and counting up Lego-bricks, you never forget differentiation.

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Bo: Thank you, Allan, for sharing with us your view on calculus. Next time at the MATHeCADEMY.net channel we look at how to deconstruct preschool mathematics.