Calculus is Hard !
- or is it ?

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Deconstruction

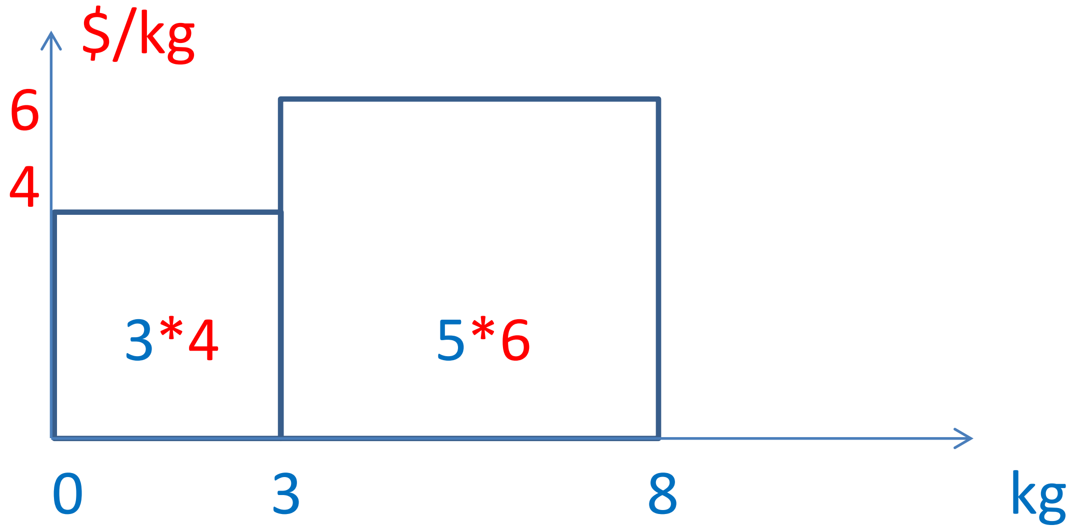
discovers alternatives to
<u>choices</u>
presented as
<u>nature</u>

Root of Calculus:

Adding PerNumbers

	3 kg	at	4 $/kg
+	5 kg	at	6 $/kg
	8 kg	at	? $/kg

Areas add PerNumbers

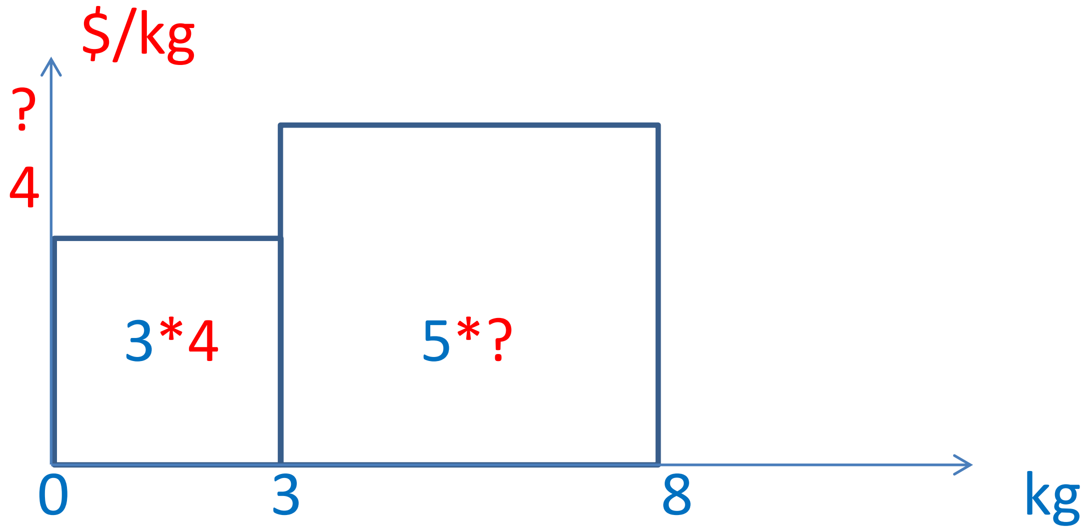


3 kg at 4 $/kg + 5 kg at 6 $/kg =

(3+5) kg at (3*4 + 5*6)/(3+5) $/kg

$\boxed{* \; +}$

Differentiation: Reversed Integration

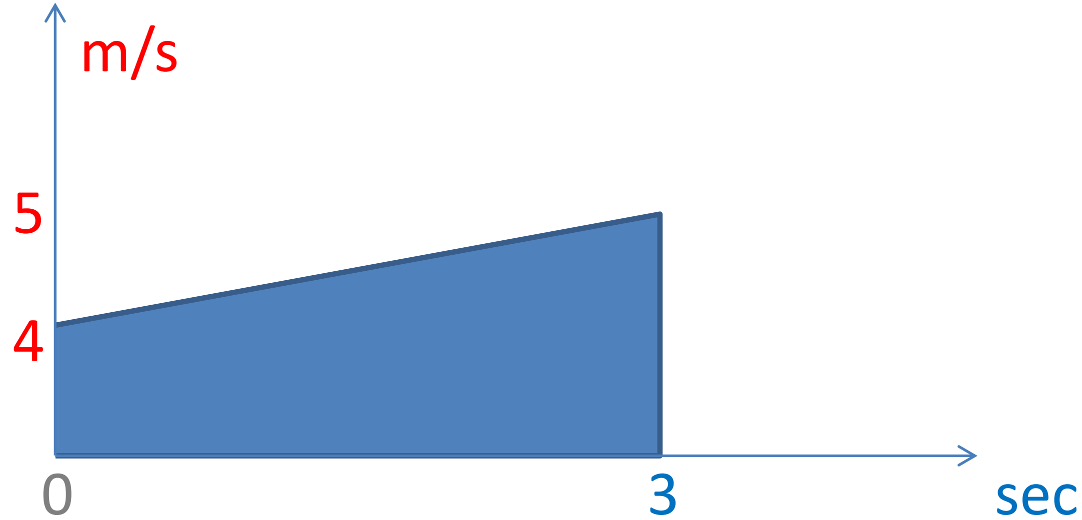


3 kg at 4 \$/kg + 5 kg at ? \$/kg = 8 kg at 7 \$/kg

? = (8*7 – 3*4)/5

– /

Changing PerNumbers



3 sec. at 4 m/s increasing to 5 m/s = ? m

Defining Constancy

y is GLOBALLY constant = c

ForAll d: distance(y,c) < d

y is PIECEWISE constant = c

Exists e, ForAll d: distance(y,c) < d within e

y is LOCALLY constant = c

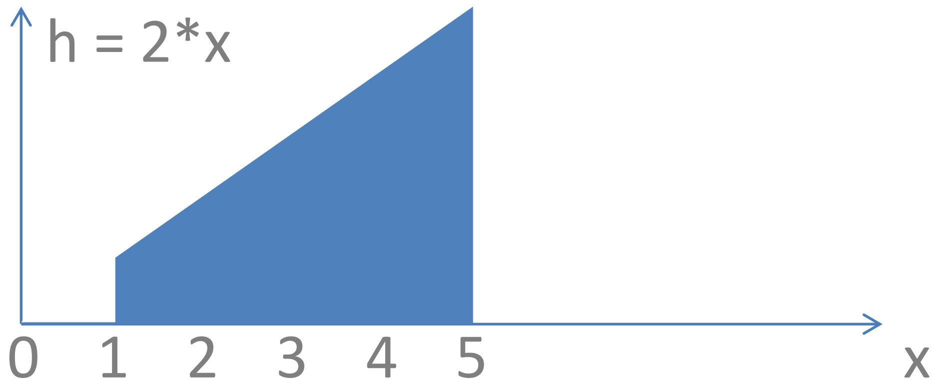
ForAll d, Exists e: distance(y,c) < d within e

Σ many changes = 1 change

y **Level**	**Δy** Single Change	**$\Sigma\Delta$y** Sum of Changes	$\Delta y = y_e - y_s$ Total Change
ys			
y1	y1 – ys	y1 – ys	y1 – ys
y2	y2 – y1	(y2 – y1) + (y1 – ys)	y2 – ys
y3	y3 – y2	(y3 – y2) + (y2 – ys)	y3 – ys
ye	ye – y3	(ye – y3) + (y3 – ys)	ye – ys

$$\int dy = \Sigma\Delta y = \Delta y = y_{end} - y_{start}$$

Area as Change



In $y = x^2$, an x-change dx gives a y-change $dy = 2*x*dx$

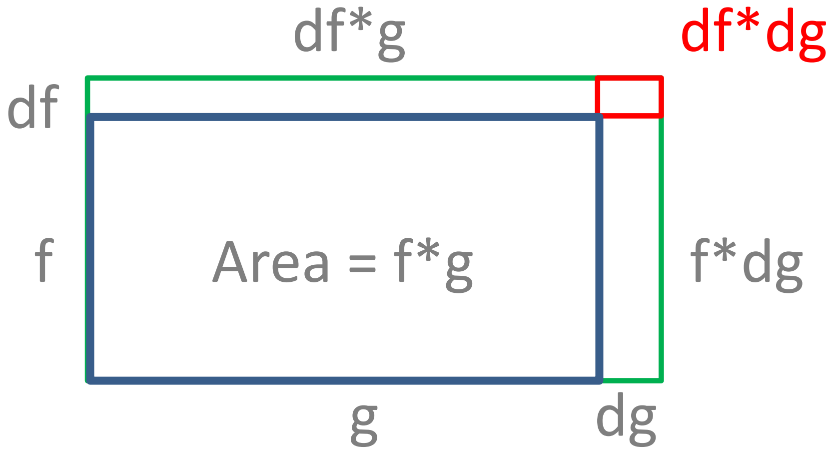
So the area A under h from x = 1 to x = 5 is

$$A = \int h*dx = \int 2*x*dx = \int dy = y5 - y1$$

$$= 5^2 - 1^2$$

$$= 24$$

The Change of a Rectangle



Change of Area = df*g + f*dg (+ df*dg)

d(f*g) = df*g + f*dg

(f*g)' = f'*g + f*g' where f' = df/dx

The Change of x^2 and x^3

If $y = x$

then $y' = dy/dx = dx/dx = 1$

If $y = x^2 = x*x$

then $y' = (x^2)' = (x*x)' = x'*x + x*x' = 1*x + x*1 = 2x$

If $y = x^3 = x^2*x$

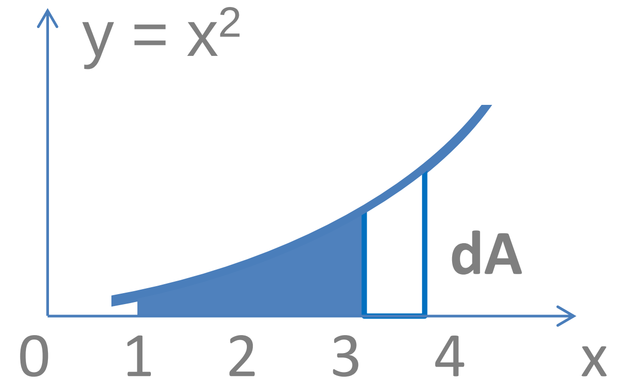
then $y' = (x^3)' = (x^2*x)' = x^{2\prime}*x + x^2*x' = 2x*x + x^2*1 = 3x^2$

Since $(f*g)' = f'*g + f*g'$

Differential Equation

Find the area under $y = x^2$ from $x = 1$ to 4

The change of the area A:
dA = y*dx , or
$A' = dA/dx = y = x^2$, $A(1)= 0$



Solution: $A = 1/3*x^3 + c$

$0 = 1/3*1^3 + c$, $c = -1/3$

so **A(4) =** $1/3*4^3 - 1/3 =$ **21**

Applying Calculus

Physics

Velocity: 3 sec at 4 m/s + 5 sec at 6 m/s = ?

Acceleration = Velocity’ = Position ’’

Economics

Price: 3 kg at 4 $/kg + 5 kg at 6 $/kg = ?

Rate of change: dy/dx = y’

Relative r. of c.: (dy/y)/dx = y’/y

Relative r. of rel. c.: (dy/y)/(dx/x) = y’/y*x

Newton: No, No, No & No

No, the moon moves not among the stars, it falls towards the earth, as does the apple.

No, they follow not the Lord's unpredictable will, they follow a formula's predictable will, a force.

No, forces give not motion, but change in motion.

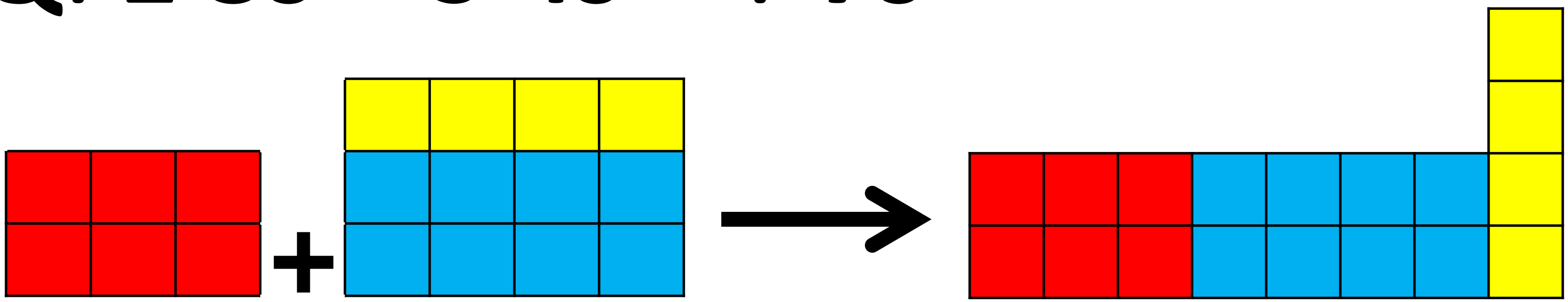
No, algebra solves not change-equations, calculus must be invented to do so.

Four Ways to Add Many

Adding	**Variable**	**Constant**
Unit-numbers	3 \$ and 5 \$ **T** = 3 + 5	3 \$ 5 times **T** = 3*5
Per-numbers	3 sec. of y m/s **T** = ∫ y dx	3 % 5 times **T** = 103%^5 - 100%

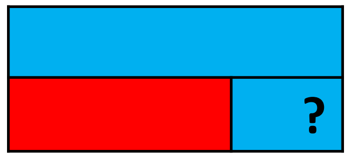
Adding NextTo

Q: 2 **3s** + 3 **4s** = ? **7s**

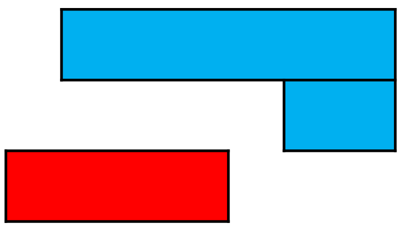
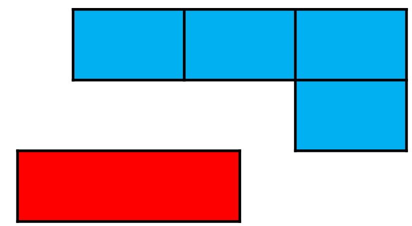


A: 2 **3s** + 3 **4s** = 2.4 **7s**

Reversed Adding NextTo



Q: 1 **4s** + ? **2s** = 2 **6s**

 first take away the 1 **4s**	 then count the rest in **2s**

A: ? = (2 **6**s – 1 **4**s)/**2** = 4 *differentiation*

Conclusion

Deconstruction discovers an alternative:
Calculus means adding per-numbers

Calculus becomes easy if built upon

- mixing quantities in middle school
- adding next-to in primary school

PreSchool Math is Hard !
- or is it ?

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Defining Linearity

y is <u>GLOBALLY</u> linear:

the per-number y’ = dy/dx is <u>globally</u> constant

y is <u>PIECEWISE</u> linear:

the per-number y’ = dy/dx is <u>piecewise</u> constant

y is <u>LOCALLY</u> linear:

the per-number y’ = dy/dx is <u>locally</u> constant

PreSchool Math

Golden learning chances

Linearity: Shift units

Integration: Add nextTo

Equations: Reverse addition